

**GENERAL
INVESTIGATIONS OF
CURVED SURFACES
OF 1827 AND 1825**

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General Investigations of Curved Surfaces of 1827 and 1825 by Karl Friedrich Gauss

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GENERAL INVESTIGATIONS

OF

"CURVED SURFACES"

OF

1827 AND 1825

TRANSLATED WITH NOTES

AND A

BIBLIOGRAPHY

BY

JAMES CADDALL MOREHEAD, A.M., M.S., AND ADAM MILLER HILTEBEITEL, A.M.
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INTRODUCTION

In 1827 Gauss presented to the Royal Society of Göttingen his important paper on the theory of surfaces, which seventy-three years afterward the eminent French geometer, who has done more than any one else to propagate these principles, characterizes as one of Gauss's chief titles to fame, and as still the most finished and useful introduction to the study of infinitesimal geometry¹. This memoir may be called: General Investigations of Curved Surfaces, or the Paper of 1827, to distinguish it from the original draft written out in 1825, but not published until 1900. A list of the editions and translations of the Paper of 1827 follows. There are three editions in Latin, two translations into French, and two into German. The paper was originally published in Latin under the title:

I a. *Disquisitiones generales circa superficies curvas*
auctore Carolo Friderico Gauss.
Societati regiae oblatae D. 8. Octob. 1827,

and was printed in: *Commentationes societatis regiae scientiarum Gottingensis recentiores, Commentationes classis mathematicae. Tom. VI. (ad a. 1823-1827). Gottingae, 1828, pages 99-146.* This sixth volume is rare; so much so, indeed, that the British Museum Catalogue indicates that it is missing in that collection. With the signatures changed, and the paging changed to pages 1-50, I a also appears with the title page added:

I b. *Disquisitiones generales circa superficies curvas*
auctore Carolo Friderico Gauss.
Gottingae. Typis Dieterichianis. 1828.

II. In Monge's *Application de l'analyse à la géométrie*, fifth edition, edited by Liouville, Paris, 1850, on pages 505-546, is a reprint, added by the Editor, in Latin under the title: *Recherches sur la théorie générale des surfaces courbes; Par M. C.-F. Gauss.*

¹G. Darboux, *Bulletin des Sciences Math.* Ser. 2, vol. 24, page 278, 1900.

III*a*. A third Latin edition of this paper stands in: Gauss, Werke, Herausgegeben von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Vol. 4, Göttingen, 1873, pages 217-258, without change of the title of the original paper (I*a*).

III*b*. The same, without change, in Vol. 4 of Gauss, Werke, Zweiter Abdruck, Göttingen, 1880.

IV. A French translation was made from Liouville's edition, II, by Captain Tiburce Abadie, ancien élève de l'École Polytechnique, and appears in Nouvelles Annales de Mathématique, Vol. 11, Paris, 1852, pages 195-252, under the title: Recherches générales sur les surfaces courbes; Par M. Gauss. This latter also appears under its own title.

V*a*. Another French translation is: Recherches Générales sur les Surfaces Courbes. Par M. C.-F. Gauss, traduites en français, suivies de notes et d'études sur divers points de la Théorie des Surfaces et sur certaines classes de Courbes, par M. E. Roger, Paris, 1855.

V*b*. The same. Deuxième Édition. Grenoble (or Paris), 1870 (or 1871), 160 pages.

VI. A German translation is the first portion of the second part, namely, pages 198-232, of: Otto Böklen, Analytische Geometrie des Raumes, Zweite Auflage, Stuttgart, 1884, under the title (on page 198): Untersuchungen über die allgemeine Theorie der krummen Flächen. Von C. F. Gauss. On the title page of the book the second part stands as: Disquisitiones generales circa superficies curvas von C. F. Gauss, ins Deutsche übertragen mit Anwendungen und Zusätzen . . .

VII*a*. A second German translation is No. 5 of Ostwald's Klassiker der exacten Wissenschaften: Allgemeine Flächentheorie (Disquisitiones generales circa superficies curvas) von Carl Friedrich Gauss, (1827). Deutsch herausgegeben von A. Wangerin. Leipzig, 1889. 62 pages.

VII*b*. The same. Zweite revidirte Auflage. Leipzig, 1900. 64 pages.

The English translation of the Paper of 1827 here given is from a copy of the original paper, I*a*; but in the preparation of the translation and the notes all the other editions, except V*a*, were at hand, and were used. The excellent edition of Professor Wangerin, VII, has been used throughout most freely for the text and notes, even when special notice of this is not made. It has been the endeavor of the translators to retain as far as possible the notation, the form and punctuation of the formulæ, and the general style of the original papers. Some changes have been made in order to conform to more recent notations, and the most important of these are mentioned in the notes.

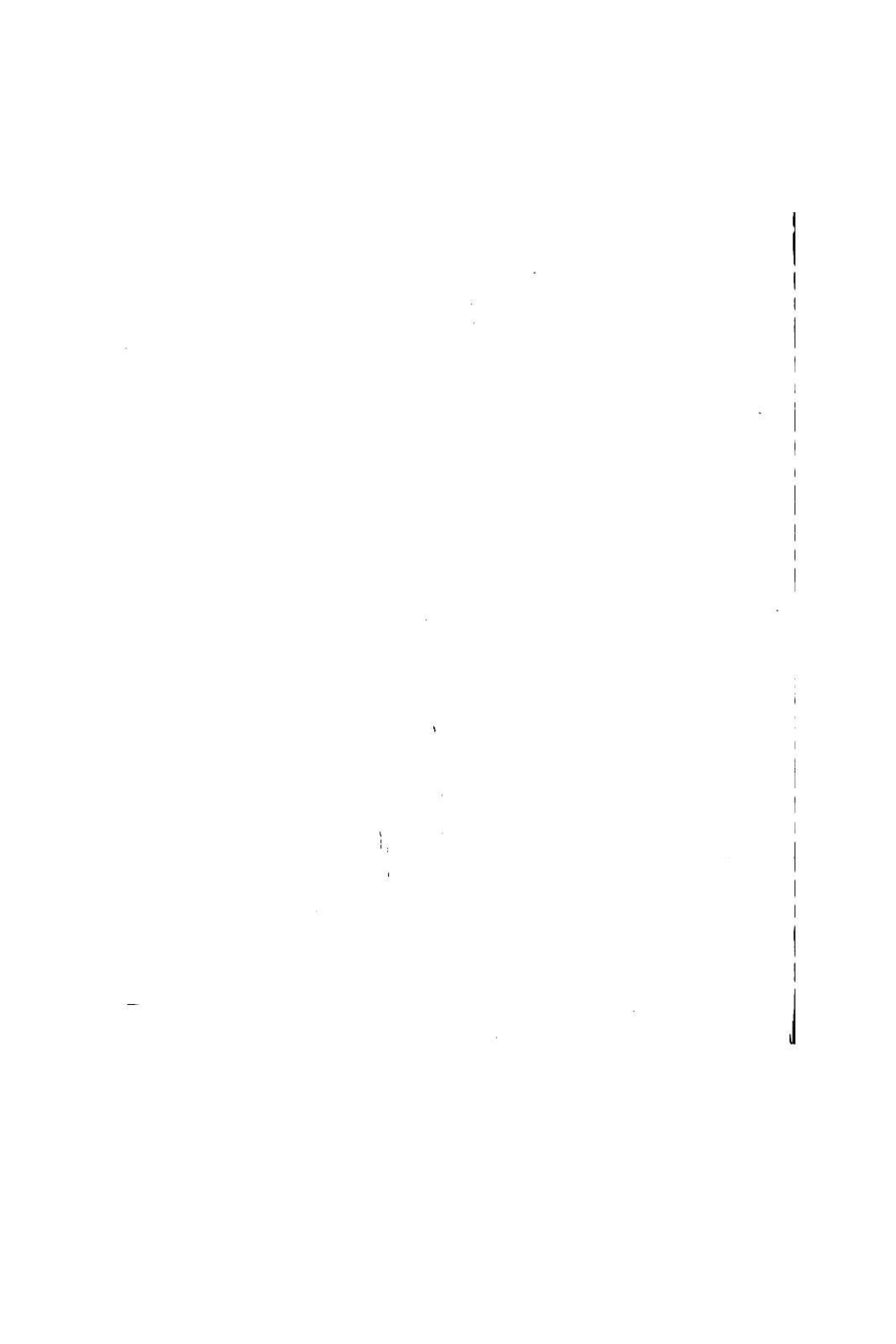
INTRODUCTION

The second paper, the translation of which is here given, is the abstract (Anzeige) which Gauss presented in German to the Royal Society of Göttingen, and which was published in the *Göttingische gelehrte Anzeigen*. Stück 177. Pages 1761-1768. 1827. November 5. It has been translated into English from pages 341-347 of the fourth volume of Gauss's Works. This abstract is in the nature of a note on the Paper of 1827, and is printed before the notes on that paper.

Recently the eighth volume of Gauss's Works has appeared. This contains on pages 408-442 the paper which Gauss wrote out, but did not publish, in 1825. This paper may be called the New General Investigations of Curved Surfaces, or the Paper of 1825, to distinguish it from the Paper of 1827. The Paper of 1825 shows the manner in which many of the ideas were evolved, and while incomplete and in some cases inconsistent, nevertheless, when taken in connection with the Paper of 1827, shows the development of these ideas in the mind of Gauss. In both papers are found the method of the spherical representation, and, as types, the three important theorems: The measure of curvature is equal to the product of the reciprocals of the principal radii of curvature of the surface, The measure of curvature remains unchanged by a mere bending of the surface, The excess of the sum of the angles of a geodesic triangle is measured by the area of the corresponding triangle on the auxiliary sphere. But in the Paper of 1825 the first six sections, more than one-fifth of the whole paper, take up the consideration of theorems on curvature in a plane, as an introduction, before the ideas are used in space; whereas the Paper of 1827 takes up these ideas for space only. Moreover, while Gauss introduces the geodesic polar coordinates in the Paper of 1825, in the Paper of 1827 he uses the general coordinates, p, q , thus introducing a new method, as well as employing the principles used by Monge and others.

The publication of this translation has been made possible by the liberality of the Princeton Library Publishing Association and of the Alumni of the University who founded the Mathematical Seminary.

H. D. THOMPSON.



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