

**NUMBERS
UNIVERSALIZED: AN
ADVANCED ALGEBRA**

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Numbers universalized: an advanced algebra by David M. Sensenig

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DAVID M. SENSENIG

**NUMBERS
UNIVERSALIZED: AN
ADVANCED ALGEBRA**

APPLETONS' MATHEMATICAL SERIES

NUMBERS UNIVERSALIZED
AN
ADVANCED ALGEBRA

BY

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PART SECOND



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PREFACE.

NUMBERS UNIVERSALIZED is believed to embrace all algebraic subjects usually taught in the preparatory and scientific schools and colleges of this country. For convenience, it is divided into two parts, which are bound separately and together, to accommodate all kinds and grades of schools sufficiently advanced to adopt its use.

Part Second is treated in five chapters, as follows : One embracing serial functions, including development of functions into series, convergency and divergency of infinite series, the binomial formula, the binomial theorem, the exponential and logarithmic series, summation of series, reversion of series, recurring series, and decomposition of rational fractional functions ; one treating of complex numbers, graphically and analytically, including fundamental operations with complex numbers, general principles of moduli and norms, and the development and representation of sine, cosine, and tangent ; one embodying a discussion on the theory of functions, including graphical representations of the meaning of the terms independent and dependent variables, continuous and discontinuous functions, increasing and decreasing functions, and turning values and limits of functions, and also a treatment of differentials and derivatives, and maxima and minima values of functions ; one treating of the theory of equations, including a discussion of the properties of the roots, real and imaginary, of an equation, methods of determining the commensurable roots of a numerical equation, Sturm's theorem for detecting the number and situation of real roots, Horner's method of root extension, Cardan's for-

mula for solving cubic equations, and a short treatment of reciprocal and binomial equations; one treating of determinants and probabilities, so far as these subjects are of interest and value to the general student. The volume closes with a supplementary discussion of continued fractions and theory of numbers.

The aim of the author in preparing this part of his work has not been so much to give completeness to the various subjects treated as to lead the student to a comprehension of the fundamentals of a wider range of subjects, and to cultivate in him a taste for mathematical investigation. It is believed that the plan adopted will give the general student a broader and more practical knowledge of algebra, and will lead to better results in a preparatory course of study for the university than would a completer treatment of fewer subjects requiring an equal amount of space in their development and more time in their mastery. While a sufficient number of examples have been placed under each head to offer opportunity for the application of the principles and laws developed, there will not be found an unnecessary multiplicity of them to retard the progress of the pupil in his onward course.

In conclusion, the author desires to acknowledge his indebtedness to the English authors, Hall and Knight, Chrystal, Aldis, Whitworth, and C. S. Smith, whose works he frequently consulted, and from which he obtained many new and valuable ideas.

DAVID M. SENSENIG.

NORMAL SCHOOL, WEST CHESTER, PA., }
December 2, 1889. }

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