LECTURES ON THE GEOMETRY OF POSITION; PART I

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GEOMETRY OF POSITION

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PART I.

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TRANSLATOR'S PREFACE.

In preparing this translation of Professor Reye's *Geometrie der Lage*, my sole object has been to place within easy reach of the English-speaking student of pure geometry an elementary and systematic development of modern ideas and methods. The increasing interest in this study during recent years has seemed to demand a text-book at once scientific and sufficiently comprehensive to give the student a fair view of the field of modern pure geometry, and also sufficiently suggestive to incite him to investigation. The recognized merit of Professor Reye's work in all these regards is my only apology for offering this translation as an attempt to satisfy our present needs.

It has been my aim to present in fair readable English the geometric ideas contained in the text, rather than to hold myself, at all points, to a literal translation; yet I trust that I have not altogether destroyed the charm of the original writing. Some changes have been made; the articles have been numbered, the examples set at the end of the lectures to which they are related and a few new ones added, explanatory notes have been inserted where they seemed necessary or helpful, and an index has been compiled. I have not deemed it advisable to omit from this edition any part of the original prefaces or introduction, even though, at this distance from their first publication, they might not be demanded in their entirety.

For the most part I have endeavoured to hold rigorously to well-established terminology. A few instances of deviation from this principle, however, may be mentioned. I have preferred the terms 'sheaf of rays,' 'sheaf of planes,' and 'bundle of rays or planes,' to the more common though I think less expressive terms, 'flat pencil,' 'axial pencil,' and 'sheaf of lines or planes'; instead of the expression 'conformal representation' as an equivalent for the German '*conforme*

Abbildung,' I have ventured 'conformal depiction.' The term 'ideal' has elsewhere been applied to infinitely distant points and lines; with this I have associated the word 'actual' to apply to points and lines of the finite region.

I desire to acknowledge my indebtedness to my colleague Professor Henry S. White for valuable assistance; my thanks also and the gratitude of all who may profit by the use of this translation are due to Dr. M. C. Bragdon of Evanston whose interest and generosity made its publication possible.

What is commonly known as Modern Synthetic Geometry has been developed for the most part during the present century. It differs from the geometry of earlier times, not so much by the subjects dealt with and the theorems propounded, as by the processes which are employed and the generality of the results which are attained.

Geometry was to the ancients a subject of entrancing interest. Its progress is prominently connected with the names of Thales of Miletus (640-546), Pythagoras (569-500), Plato (429-348), who cultivated geometry as fundamental to the study of philosophy, Menaechmus (375-325), the first to discuss the conic sections, Euclid of Alexandria (330-275), Archimedes (287-212), and Apollonius of Perga (260-200); these among others before the Christian era.

Of the numerous writings of Euclid, the *Elements*,[1] in which was collected and systematized much of the geometrical knowledge of that time, has remained for two thousand years a marvellous monument to his skill. Whatever may be its defects, and these have been the subject of much discussion, it "certainly possesses some excellent features; it accustoms the mind to rigor, to elegance of demonstration, and to the methodical arrangement of ideas; in these regards it is worthy of our admiration."[2] His *Porisms*, which unfortunately have been lost, are said to have contained many of the principles that have formed the basis of modern geometry.

Ancient geometry reached its highest perfection under Archimedes and Apollonius, the former of whom devoted much study to physical problems by means of geometry, and the latter carried his investi-

[1] For a convenient summary and characterization of Euclid's *Elements*, see Professor Henrici's article on Geometry in the *Encyclopedia Britannica*.

[2] Poncelet, *Propriétés Projectives*, etc., p. 15.

gations upon the conic sections so far as to leave few of their important properties undiscovered. He produced a systematic treatise on conic sections containing his own discoveries, and including also all previous knowledge of these curves.[1]

The great geometer and commentator of the early centuries of the Christian era was Pappus of Alexandria. In his *Mathematicae Collectiones*, written toward the end of the fourth century, he collated the scattered works of the earlier celebrated geometers and a multitude of curious theorems from many sources, to which he added so much of original work as to place him among the most illustrious of ancient geometers. This work is the chief source of information on ancient geometry. It comments so fully upon Euclid's book of porisms that frequent efforts have been made to restore the latter, notably by Chasles in 1860.

The work of the ancient geometers was fragmentary. Truly remarkable discoveries were made, but general principles were not brought into prominence; theorems were announced disconnectedly as though they had been received by their authors ready made; the method of their discovery was rarely, if ever, indicated; the demonstrations were given in the most polished and systematic form, but the relations existing among different theorems were not shown, and no suggestions were offered for further investigation; special cases of general theorems were as a rule treated as though they were separate and independent theorems.

But, scattered here and there, throughout the great volume of geometrical knowledge accumulated by these early geometers, is to be found the material upon which the beautiful and symmetrical structure of modern geometry has been founded. For example, the property of perspective triangles of which use is made in the geometrical definition of harmonic points, though usually credited to Desargues, was in fact announced by Euclid.[2] Harmonic division itself was known to Apollonius, and the fact that the anharmonic ratio of four collinear points is unaltered by projection was demonstrated by Pappus,[3] and was probably known much earlier. The theorem upon which Carnot based the theory of transversals was discovered and published by Menelaus early in the second century.

[1] An edition of Apollonius' *Conic Sections*, with notes, etc., by T. L. Heath, M.A., has recently been published by Macmillan & Co., London.

[2] Pappus, *Mathematicae Collectiones*, preface to book VII.

[3] *Mathematicae Collectiones*, VII., 129.

As has already been suggested, modern geometry is characterized by generality both in its processes and in its results. The founding of modern pure geometry is usually accredited to Monge (1746-1818), whose lectures in the Polytechnic School at Paris were published under the title of *Géométrie Descriptive*. These lectures, by utilizing the theory of transversals and the principle of parallel projection, called attention to the advantages to be gained through the application of geometrical methods, and served to revive the interest in pure geometry, which had been dormant for so many years.

But the generalizing processes which characterize modern geometry were begun much earlier than the time of Monge. Desargues (1593-1662), a contemporary of Descartes, introduced the notion of infinitely distant points and lines, with its far reaching results, and announced the doctrine of continuity. The methods of Pascal (1623-1662) too, so far as it is possible to judge from the few remaining fragments of his mathematical work, partook of the broadest generality, and it is fair to assume that had not the work of these two great geometers been almost entirely lost, and had not their ideas been wholly pushed aside through the overwhelming influence of Descartes' discoveries, many of the geometrical theories and results of the present century would have been developed long ago, and the so-called modern geometry would have been of much earlier date.

As it was, however, pure geometry was but little cultivated for over a hundred years before the time of Monge. Geometrical knowledge was truly increased during this period, especially by Newton (1642-1727), Maclaurin (1698-1746), Robert Simson (1687-1768), and Matthew Stewart (1717-1785), but their methods could scarcely be said to partake of the spirit of modern geometry, and differed but little, if at all, from the methods of the ancient geometers.

The illustrious names in connection with the development of modern pure geometery are Poncelet (1788-1867), Steiner (1796-1863), Von Staudt (1798-1867), and Chasles (1793-1880); and if it were permissible to add the names of living men I should mention Cremona and Reye.

Poncelet's great work, *Traité des propriétés projectives des figures*, etc., appeared in 1822, and at once clearly justifies any claim that may be set up in his behalf as the leader in the so-called modern methods. In this work the principle of continuity, the principle of reciprocity or duality, and the method of projection are the chief factors.