LESSONS ON RIGID DYNAMICS

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RIGID DYNAMICS,

BY THE

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PREFACE.

It will be generally acknowledged, I think, that there is no subject of Natural Philosophy, equal in importance to that familiarly known as Rigid Dynamics, of which the study is so exclusively restricted to the more advanced students of Mathematics. Yet this restriction cannot be said to be necessary, for the treatment of the subject involves none of the higher mathematical methods; and it must be allowed to be unfortunate, for the science of motion is the basis of Mechanical Engineering, and furnishes the explanation of many interesting terrestrial and cosmical phenomena.

This restriction of the study is chiefly due to the fact that, while the conceptions and reasoning peculiar to the subject are somewhat difficult, the explanations of its leading principles, given in the books commonly used by students, are for the most part very brief, and often, through brevity, obscure.

It is this deficiency of explanation which I have attempted to supply in the following little book. It is not my purpose to acquaint the student with the splendid generalizations of Lagrange and of more recent philosophers. For that the books in present use leave nothing to be desired. My aim is to render more general the study of this interesting science, by presenting as simple a view of its principles as is consistent with scientific accuracy, and to give a sound foundation to the student who is to proceed higher.

It is my hope that the book may be useful not only to students of Natural Philosophy, but also to engineers. Most of them possess a knowledge of the principles of Mechanics, of the method of Co-ordinate Geometry, and of the Integral Calculus; and that is all that is here required.

The principle on which this science is based has been so long connected with the name of D'Alembert that it would hardly be recognised under any other. Nevertheless there is no doubt that Euler has more claim to its authorship, inasmuch as he first used it. D'Alembert admits this, but says that Euler gave no proof. I believe D'Alembert's real merit to be, that his explanation was exactly suited to clear away the difficulties which were perplexing men's minds.

The works to which I am principally indebted are:— Thomson and Tait's *Natural Philosophy;* Routh's *Rigid Dynamics;* Resal's *Cinématique Pure;* Rankine's *Machinery and Millwork;* Walton's *Mechanical Problems;* Whewell's *History of the Inductive Sciences;* Willis' *Principles of Mechanism;* Müller's *Lehrbuch der kosmischen Physik;* Montucla's *Histoire des Mathématiques;* D'Alembert's *Traité de Dynamique,* and Euler's *Mechanik.*

My thanks are due to Dr Campion, of Queens' College, for many valuable suggestions which he has made; and to several of my pupils for their frank statement of their difficulties.

G. PIRIE.

QUEENS' COLLEGE, CAMBRIDGE.
December, 1874.

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GEOMETRY OF MOTION.

I.

1. A RIGID body is an assemblage of particles such that the distance between each pair is unchangeable. The movements of such a body are very different from those of a set of independent points. Its fixed connections introduce a common movement. Any straight line or any plane of particles in the body must remain always a straight line or plane. If all such planes remain parallel to themselves, the motion is one of translation. But if any such plane makes an angle with its former position the motion is rotational. And the velocity of rotation—angular velocity—is measured by the rate at which the plane is describing angles.

Thus the connecting rod of the driving wheels of a goods' locomotive has only a translational motion;—so also (approximately) the axis of the earth in its yearly motion round the sun. In a well-thrown quoit the motions are combined.

2. From this definition of rotation it follows that a point cannot rotate. It may revolve about another point, but it contains no lines nor planes which can describe angles. For rotation there must be an extended system. A point in motion may be said to be revolving about any point whatever situated in the line through it at right angles to its direction of motion, for it is moving at the moment in a circle with the point as centre. But the body of which this is a point may not be rotating. For rotation it is necessary that the different points of the body should be at the moment revolving about the same axis.