MATHEMATICAL QUESTIONS WITH THEIR SOLUTIONS, FROM THE "EDUCATIONAL TIMES"; VOL. XLI

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Mathematical Questions with Their Solutions, from the "Educational Times"; Vol. XLI by $\,$ W. J. C. Miller

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W. J. C. MILLER

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EDITED BY

W. J. C. MILLER, B.A.,

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1945. (The late C. W. Merrifield, F.R.S.) — Find a rectangular parallelepiped such that its edges, the diagonals of its faces, and the diagonals of the solid, shall all be integral
3835. (The Editor.)—The sides of a triangle ABC are BC = 6, CA = 5, AB = 4; and Q, R are points in AC, AB, such that $CQ = 2$; BR = 3. Show (1) by a general solution, that the distance from B to point P in BC, such that $\angle CQP = BRP$, is BP = $\frac{1}{2}(301^{3}-13) = 3.83843$ and (2) give a construction for finding the point P.
3873. (J. B. Sanders.)—The horizontal section of a cylindrica vessel is 100 square inches, its altitude is 36 inches, and it has an orifice whose section is 10 of a square inch; find in what time, if filled with a fluid, it will empty itself, allowing for the contraction of the vein 123
4516. (The late T. Cotterill, M.A.)—In a spherical triangle, of the five products
cos a cos A, cos è cos B, cos e cos C, cos a cos è cos e, -cos A cos B cos C
one is negative, the other four being positive. In the solution of sucl triangles, what parts must be given that the affections of the remaining three can be determined by this theorem?
4925. (The late Professor Clifford, F.R.S.)—Let U, V, W = 0 be the point equations, and u , v , $w = 0$ the plane-equations of thre quadrics inscribed in the same developable, and let $u+v+w$ be identically zero. Then, if a tangent plane to U, a tangent plane to V, and a tangent plane to W, are mutually conjugate in respect of $u + bv + cw = 0$
they will intersect on $\frac{\mathbf{U}}{(b-c)^2} + \frac{\mathbf{V}}{(c-a)^2} + \frac{\mathbf{W}}{(a-b)^2} = 0,$
which passes through the curves of contact of the developable with ss + bv + cw and one other quadric
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5350. (S. A. Renshaw.)—An ellipse and hyperbola have the sam
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centre and directrices, and they have a common tangent which touches the ellipse in D and the hyperbola in E, and meets one of the directrices in H. Also from the common centre of the curves S'R is drawn parallel to the common tangent and meeting the same directrix in R. Tangents RW, RV are drawn to the auxiliary circles of the ellipse and hyperbola. Show that, if FH, fH be joined, F and f being the foci of the curves belonging to the directrix RH,

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5421. (Professor Cayley, F.R.S.)—Suppose $S_n=m_1\ (x-a_1),\ m_2\ (x-a_2),\ m_3\ (x-a_3),\ m_4\ (x-a_4)$; where, for any given value of x, we write +, -, or 0, according as the linear function is positive, negative, or zero, and where the order of the terms is not attended to. If x is any one of the values $a_1,\ a_2,\ a_3,\ a_4$, the corresponding S is 0+++, 0---, 0++-, or 0+--; and if I denote indifferently the first or second form, and R denote indifferently the third or fourth form, then it is to be shown that the four S's are R, R, R, R, or else R, R, I, I.

6754. (J. Hammond, M.A.)—Sum the series $\frac{1}{n} \cdot \frac{1}{2m+n} - 2m \cdot \frac{1}{n+1} \cdot \frac{1}{2m+n-1} + \frac{2m \cdot (2m-1)}{1 \cdot 2} \cdot \frac{1}{n+2} \cdot \frac{1}{2m+n-2} = \delta c.,$

where m is a positive integer, and the (r+1)th term is

$$(-)^{r} \frac{2m(2m-1)\dots(2m-r+1)}{1\cdot 2\cdot 3\dots r} \cdot \frac{1}{n+r} \cdot \frac{1}{2m+n-r} \cdot \dots 32$$

5787. (W. J. C. Sharp, M.A.) — From an ordinary point on a quartic five straight lines can be drawn so as to be cut harmonically by two curves. How far is this modified when the point is a node? 31

6063. (The Rev. A. J. C. Allen, B.A.)—A prism filled with fluid is placed with its edge vertical, and a beam of light is passed through an infinitely thin vertical slit, and is incident normally on the prism infinitely near its edge. The emergent beam is received on a vertical screen. If the refractive index of the fluid varies as the depth below a horizontal plane, find the nature and position of the bright curve formed in the screen.

6907. (S. Tebay, B.A.)—If A, B, C can do similar pieces of work in s, b, c hours respectively, (a < b < c); and they begin simultaneously, and regulate their labour by mutual interchanges at certain intervals, so that the three pieces of work are finished at the same time: find the number of solutions.

7040. (Rev. T. R. Terry, F.R.A.S.)—If p and q be two positive integers such that p>q, and if r be any positive integer, or any negative integer numerically greater than p, show that

$$1 - \frac{q}{p - q + 1} \cdot \frac{r}{p + r - 1} + \frac{q(q - 1)}{(p - q + 1)(p - q + 2)} \cdot \frac{r(r - 1)}{(p + r - 1)(p + r - 2)} - \&c.$$

$$= \frac{p - q}{p} \cdot \frac{p + r}{p - q + r}.$$
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7155. (T. Woodcock, B.A.)—If P, Q be the points in which the plane through the optic and ray axes intersects the circle of contact PQ of a tangent plane perpendicular to an optic axis of the wave surface of a biaxal crystal, and if a, c, the greatest and least axes of clasticity, be given; prove that, O being the centre of the wave surface, (1) the triangle POQ, (2) the circle of contact PQ, (3) the angle POQ will have their greatest values respectively, when the square of the mean axis b is ii.) the arithmetic, (ii.) the geometric, (iii.) the harmonic mean of a² and b²; and the cone whose vertex is O and base the circle PQ will have its maximum volume when b³ = \(\frac{1}{2} \) \

7169. (R. Knowles, B.A., L.C.P.) — In a parabola whose latus rectum is 4a, if θ be the angle subtended at the focus S by a normal chord PQ, prove that the area of the triangle $SPQ = a^2 \cot \frac{1}{2}\theta \left(\tan \frac{1}{2}\theta + 4 \cot \frac{1}{2}\theta\right)^2$.

7194. (Professor Wolstenholme, M.A., Sc.D.)—In the examination for the Mathematical Tripes, January 2, 1868, Question (6) is as follows:—"If there be n straight lines lying in one plane so that no three meet in one point, the number of groups of n of their points of intersection, in each of which no three points lie in one of the n straight lines, is $\frac{1}{2}(n-1)$." Prove that this is not true; but that, if "n-sided polygons" be written for "groups of n points, &c.," the result will be true: and calculate the correct answer to the question enunciated. ... 57

7230. (The Editor.)—On a square (A) of a chess-board, a knight is placed at random: find the probability that it can march (1) from that square (A) to a given square (B), as, for example, to one of the corner-squares, within a moves; and (2) over \(\delta \) aquares in less than \(\ext{emoves} \), for instance, over the four corner-squares of the board.

7247. (Dr. Curtia.) — Two magnets, whose intensities are \mathbf{I}_1 , \mathbf{I}_2 , and lengths a_1 , a_2 , are rigidly connected so as to be capable of moving only in a horizontal plane round a vertical line, which passes through the middle point of the line connecting the two poles of each magnet; if 2a denote the angle between the lines of poles of the two magnets in the