

**BRITISH OFFICES LIFE TABLES, 1893: NET
PREMIUMS AND VALUES (IN TERMS OF
AN ASSURANCE OF 1000). BASED UPON
THE GRADUATED EXPERIENCE OF WHOLE
LIFE PARTICIPATING ASSURANCES ON
MALE LIVES ON 3 AND 3 1/2 PER CENT**

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British Offices Life Tables, 1893: Net Premiums and Values (in Terms of an Assurance of 1000).
Based Upon the Graduated Experience of Whole Life Participating Assurances on Male Lives
OM 3 and 3 1/2 Per Cent by Thos. Bradshaw & Frank Sanderson

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THOS. BRADSHAW & FRANK SANDERSON

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Net Premiums and Values

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BASED UPON THE

GRADUATED EXPERIENCE OF

WHOLE LIFE PARTICIPATING ASSURANCES

ON

MALE LIVES

O^M

3 and 3½ per cent. Interest

INCLUDING

Commutation and Valuation Columns.

By

THOS. BRADSHAW, F.I.A.

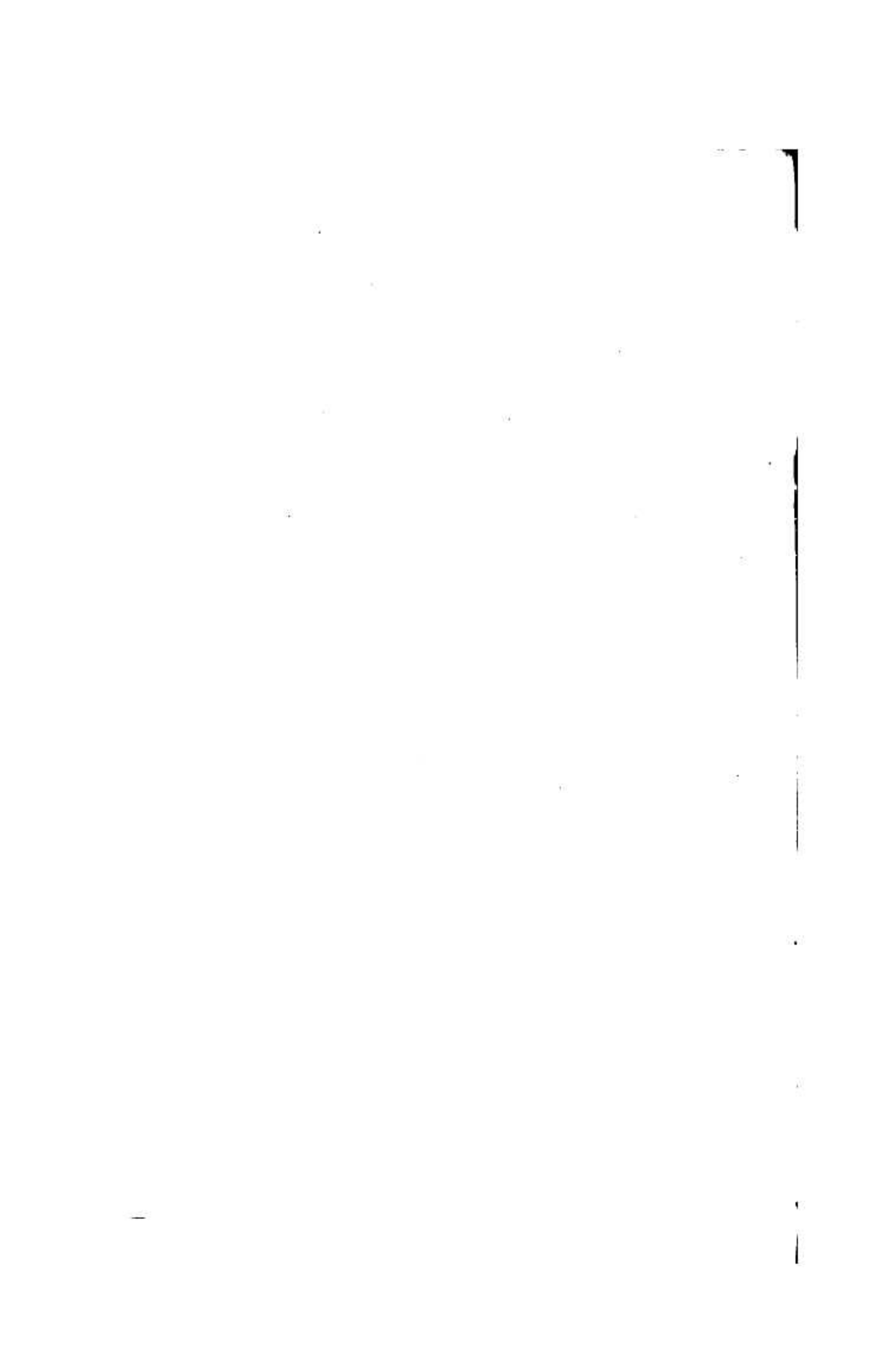
AND

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The Insurance Institute of Toronto

TORONTO, CANADA

1905



INTRODUCTION

The present volume owes its origin to the belief that the O^M Mortality Table will become, in Great Britain and the Colonies at least, the standard table for valuation of life assurance contracts, the determination of surrender values and the distribution of surplus.

Just as the H^M Mortality Table gradually superseded the older Seventeen Offices' or Actuaries' Table and the still older Carlisle Table, so the O^M Table, the most modern and scientifically constructed table now available for general life assurance purposes, is destined to supersede the H^M Table. Even where it is not adopted for official use, no company can well afford to leave its actuaries uninformed as to the nature of this new experience and the important place it is to fill in the actuarial departments of many of the best managed companies in the world. The results brought out in this volume make it possible for interesting comparisons to be instituted with other standard mortality tables.

The title of the work indicates its principal contents—net premiums and net values deduced from the graduated experience of Whole Life Participating Assurances on Male Lives—British Offices Life Tables, 1893, with interest both at 3 and at 3½ per cent.

The premiums and values have been tabulated in terms of an assurance of 1000. The tables have been printed and arranged for practical office use and each table has its contents clearly set forth without recourse to symbols. The Endowment Assurance tables, it will be noticed, have been constructed in respect to assurances payable at the end of certain periods rather than at specified ages. In the United States and Canada, Endowment Assurances are almost wholly of the first mentioned form.

Towards the close will be found the fundamental tables, viz.—the Mortality Table; Present Value to 10 decimal places of 1 due at the end of any year from 1 to 100, at 3 and 3½ per cent. interest; Commutation Columns D_x , N_x , C_x and M_x ; and Valuation Columns $u_x = \frac{D_x}{D_{x+1}}$ and $k_x = \frac{C_x}{D_{x+1}}$

The tables were obtained and formed as follows:

Mortality Table—The l_x and d_x columns were taken from "The British Offices Life Tables, 1893," published in 1902 by The Institute of Actuaries and The Faculty of Actuaries in Scotland.

Commutation Columns—The D_x and C_x columns were constructed by means of a 16 figure arithmometer, each value being calculated independently by actual multiplication. An interest factor, v^n , taken to 10 decimal places was employed. The N_x and M_x columns were formed by addition. The N_x column is of the "initial" form, namely, $N_x = D_x + D_{x+1}$, etc. All the values in the Commutation Columns have been taken to 3 more decimal places than the corresponding values in "The British Offices Life Tables, 1893."

Valuation Columns—The column $k_x = \frac{C_x}{D_{x+1}}$ was first constructed by dividing d_x by l_{x+1} on the arithmometer, the quotients k_x being recorded to 8 decimal places. The columns $u_x = \frac{D_x}{D_{x+1}}$, at 3 and $3\frac{1}{2}$ per cent. interest were then formed from the column k_x by adding unity to each value and multiplying first by 1.030 for the 3 per cent. value and then, without clearing the machine, by .005 for the $3\frac{1}{2}$ per cent. value.

Net Premiums and Values—The arithmometer was employed throughout in the construction of the tables of net premiums and values. In respect to the net single premiums for Whole Life Assurance and the net annual premiums for all systems excepting Joint Life Assurance, each premium was calculated independently from the Commutation Columns. The table of net single premiums for Endowment Assurance was formed by first calculating the single premium at the youngest age, 20, for all periods, and at the greatest period, 40, for all ages, from the Commutation Columns, and then filling in the remainder of the table by the continued formula $A_{x+(n-1)} = A_{x:n} \times u_x - k_x$. The net annual premiums for Joint Life Assurance were obtained by utilising the Joint Life Annuity values published in "The British Offices Life Tables, 1893": from the reciprocals of these values the constant d was subtracted. The net values for all systems and durations were obtained by the continued process employing the formula ${}_{t+1}V_x = ({}_tV_x + P_x) u_{x+t} - k_{x+t}$, the annual premiums used with this formula being taken to 8 decimal places in terms of an assurance of unity.

It was recognized that in many cases the premiums and values could be obtained more conveniently than by the methods described. For example, in calculating the premiums and values for many of the systems of assurance the arithmometer might have been employed to greater advantage in conjunction with tables of Whole Life and Temporary Annuities. It was thought advisable, however, to calculate all the original premiums and values as indicated, reserving any shorter or more convenient methods for the computation of a duplicate set of premiums and values for verification purposes.

In cutting down the premiums and values to the 5 decimal places, in terms of an assurance of unity, which appear in the published results, the final figures were adjusted in each case upon the basis of there being 7 decimal places, in terms of an assurance of unity, in the premiums and values of the working sheets.

Verification—After the printer's copy had been prepared, the entire work was re-calculated by new operators without reference to the original figures, a different method, as before mentioned, being employed for the calculation of the premiums and values in many cases. The printer's copy, before being set up, was called over with this second set of premiums and values, and any differences were carefully investigated. The first press proof was also called over with the second

set of premiums and values. The final proof was called over with the original set.

It may also be stated in connection with the accuracy of the results that on computing the first set of values by the continued process referred to, the final values in the Limited Payment Life and Limited Payment Endowment Assurance systems were carefully compared with the single premiums corresponding to these values. In the Continued Payment Endowment Assurance and Term Assurance systems, the extent to which each final value differed from 1000 or 0 respectively was carefully noted. In forming the tables of net values for Ordinary Whole Life Assurance, and single premiums for Endowment Assurance, verification and adjustment values were calculated separately from the Commutation Columns and inserted for each age at intervals of 10 years. With these values those obtained by the continued process were compared. In none of the comparisons was there revealed an error greater than .00000075 in terms of an assurance of unity. The average error in the final values at all ages varied from .00000001 approximately in respect to the systems of shortest duration to .00000025 approximately in respect to those of longest duration.

In order to ensure the utmost accuracy possible with the use of a 16 figure arithmometer, an unusually large number of decimal places were made use of in the formation of the tables. To this, perhaps, more than to a different method of computation, are due the differences which in some instances will be found in the final figures of the premiums for Whole Life Assurance as printed in "The British Life Offices Tables, 1893," and as printed in this volume. Also the lack of complete agreement between the Commutation Columns contained in "The British Life Offices Tables, 1893," and the extended Commutation Columns contained in this volume, will be found to arise, it is believed, from the same cause.

The authors do not desire it to be inferred that the net premiums and values tabulated are the most suitable for all the systems of assurance contained in the volume. This remark has special reference to the tables containing premiums and values for Term Assurances. But it was decided to make the work complete by including net premiums and values for all systems of assurance in common use.

The authors take this opportunity of gratefully acknowledging their indebtedness to Mr. G. Cecil Moore, A.I.A., for valuable assistance he has afforded in connection with the work.

In conclusion, the authors would express their high appreciation of the service rendered by The Institute of Actuaries of Great Britain and The Faculty of Actuaries in Scotland in giving to the life assurance world the invaluable data upon which the tables in this work are founded. The authorities of these two influential bodies have very kindly granted permission to reprint in this work the l_x and d_x columns from the official O^M Table.

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