

**A TREATISE ON  
ANALYTICAL  
STATICS, WITH  
NUMEROUS EXAMPLES**

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A Treatise on Analytical Statics, with Numerous Examples by Edward John Routh

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**EDWARD JOHN ROUTH**

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ANALYTICAL STATICS

WITH NUMEROUS EXAMPLES

BY

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## PREFACE.

**I**N the first volume of this Treatise the subjects of Attractions, Bending of Rods and Astatics were omitted. These form the substance of this volume, which therefore completes the work.

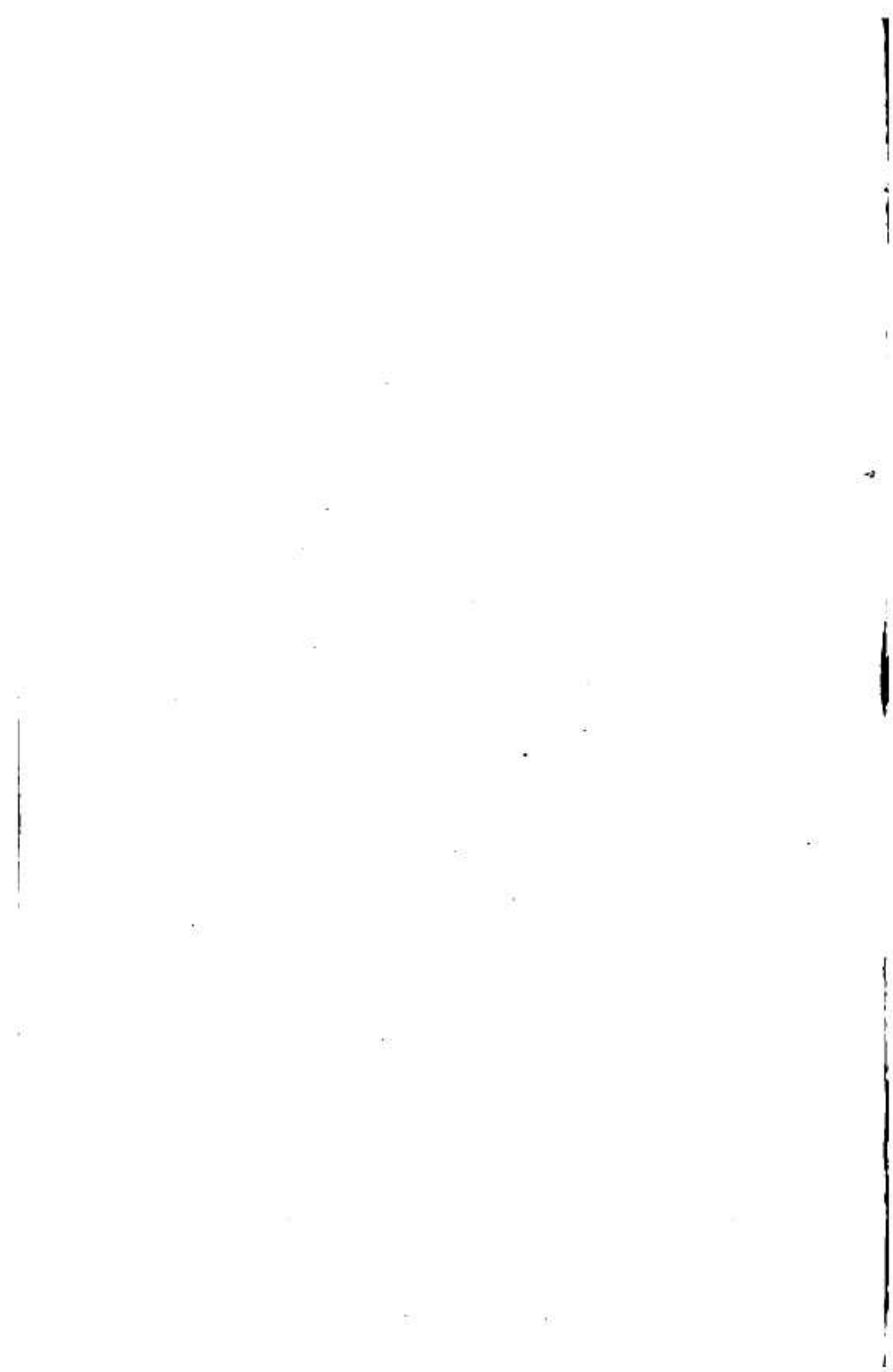
As so many isolated theorems are included under the head of Attractions, it has been thought necessary to add an Index. While the table of Contents has been made full enough to give a general view of the whole subject, it is hoped that the index will enable the reader to find easily any proposition he is seeking for. Though less necessary, indices to both the other parts of this volume have also been given.

As in the first volume, care has been taken to refer each result to its original author. Many examples also have been taken from the various examination papers set in the University.

I again desire to express my thanks to Mr E. G. Gallop of Gonville and Caius College for the very great assistance he has given me in correcting most of the proof-sheets, and for his many valuable suggestions.

EDWARD J. ROUTH.

PETERHOUSE,  
*September 3, 1892.*





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