

**INDUCTIVE PLANE GEOMETRY,
WITH NUMEROUS EXERCISES,
THEOREMS, AND PROBLEMS
FOR ADVANCE WORK**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649112340

Inductive plane geometry, with numerous exercises, theorems, and problems for advance work
by G. Irving Hopkins

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

G. IRVING HOPKINS

**INDUCTIVE PLANE GEOMETRY,
WITH NUMEROUS EXERCISES,
THEOREMS, AND PROBLEMS
FOR ADVANCE WORK**

INDUCTIVE
PLANE GEOMETRY

WITH

NUMEROUS EXERCISES, THEOREMS, AND PROBLEMS
FOR ADVANCE WORK

BY

G. IRVING HOPKINS

INSTRUCTOR IN MATHEMATICS AND ASTRONOMY
HIGH SCHOOL, MANCHESTER, N.H.

REVISED EDITION

BOSTON, U.S.A.

D. C. HEATH & CO., PUBLISHERS

1902

PREFACE.

THE inductive method of teaching geometry is recognized as the ideal one by all progressive teachers. Committing to memory the demonstration of another is acknowledged to be of little benefit to the pupil. Development of the reasoning powers comes only from their judicious exercise; and this is possible only in small degree from merely yielding assent to the logic of another.

In an experience of twenty years the author has found that fully three fourths of his pupils can demonstrate unaided, or at most with a suggestion or two, the majority of the theorems, the demonstrations of which are given in most text-books for the pupil to read and memorize. Consequently he has endeavored to be consistent, and offer aid in the way of suggestions only where the pupil needs it. In most text-books the proof of the following easy theorem is given for the pupil to memorize; viz.: "If two parallel lines are cut by a third straight line the exterior interior angles are equal." On the very same page are given others, far more difficult, for the pupil to prove unaided. The same condition of things is found on many subsequent pages. The author's experience has been that nine tenths of his pupils demonstrate the above theorem correctly unaided, and that the other tenth need only a single suggestion. The same is true of a large number of the theorems and construction problems usually given in full.

In this revised edition, the arrangement of the sequence of theorems has been radically changed in many places, noticeably in placing the subject of triangles as near the beginning as possible. This is done because of the great advantage in the

use of equality of triangles in subsequent demonstrations. This aids in reducing to a minimum the method of superposition which usually confuses the beginner.

The author has endeavored to remedy the defects of the old edition, and in this has been materially aided by various suggestions from numerous teachers who are in sympathy with the plan, and to whom he takes this occasion to express his thanks. Thanks are also due the publishers for the excellence of the mechanical work, as well as for many suggestions as to arrangement.

Finally, one of the most important features, in the estimation of the author, and one which he hopes will commend itself to teachers, is the printing the *essential theorems* in different type from the rest, so that a glance at the page will disclose them; *e.g.* :

If two sides and included angle of one triangle are equal respectively to two sides and included angle of another, the two triangles are equal.

Many of the others are helpful in proving subsequent theorems, while all are useful as exercises.

G. I. H.

MANCHESTER, N. H.,
June, 1902.

CONTENTS.

	PAGE
DEFINITIONS	1
GENERAL AXIOMS	7
PARTICULAR AXIOMS	8
SYMBOLS	12
ABBREVIATIONS	12
THEOREMS	14
TRIANGLES	16
TRANSVERALS	21
ANGLE MEASUREMENT	32
ADVANCE THEOREMS	38, 68, 97
QUADRILATERALS	35
CIRCLES	42
RATIO AND PROPORTION	53
THEORY OF LIMITS	61
PROPORTIONAL LINES	70
SIMILAR FIGURES	74
POLYGONS	78
PROBLEMS OF COMPUTATION	82, 100, 121
PROJECTION	84
AREAS	84

	PAGE
MEASUREMENT OF THE CIRCLE	105
PROBLEMS OF CONSTRUCTION	126
MISCELLANEOUS PLANE PROBLEMS FOR ADVANCED WORK :	
I. Triangles	147
II. Quadrilaterals	150
III. Circles	151
IV. Transformation of Figures	151
V. Division of Figures	154
SOLUTIONS BY ALGEBRA	159
SPECIAL THEOREMS	164
MAXIMA AND MINIMA	176
SYMMETRY	185
THEOREMS ON SYMMETRY	187
TABLES OF METRIC AND COMMON MEASURES	189
EXAMINATION QUESTIONS	191

PLANE GEOMETRY.



INTRODUCTION.

1. **Space** is indefinite extension in every direction.

2. **A Material Substance** is anything, large or small, solid, liquid, or aeriform, visible or invisible, that occupies space.

It therefore follows that material substances have *limited extension* in every direction.

3. For purposes of measurement, extension in *three* directions only are considered, called, respectively, *length*, *breadth*, and *thickness*; they are also called, collectively, *dimensions*.

4. **Magnitude**, in general, means size, and is applied to anything of which greater or less can be predicated, as time, weight, distance, etc.

A Geometrical Magnitude is that which has one or more of the three dimensions, as lines, angles, etc.

5. **A Geometrical Point** has position merely; *i.e.* it has no magnitude.

The dots made by pencil and crayon are *called* points, but they are really small substances used to indicate to the eye the location of the geometrical point.

6. **A Geometrical Line** has only one dimension; *i.e.* length.

The lines made by pencil and crayon are substances, and may be called *physical lines*, which serve to show the position of the geometrical lines.

7. A Straight Line is one that lies evenly between its extreme points.

This is the definition as given by Euclid. The majority of modern geometers, however, have substituted the following as stated by Newcomb, viz.:

"A *straight line* is one which has the same direction throughout its whole length."

Each is designed to express the idea of straightness, and not to convey it; for it is assumed that the idea already exists in the pupil's mind prior to the beginning of this study.

A *straight line* may also be defined as an *undeviating line*.

8. A Curved Line, or simply *curve*, is one no part of which is straight.

9. Material substances have one or more faces which separate them from the rest of space. These faces are called *surfaces*, and have, obviously, only two dimensions; *i.e.* *length* and *breadth*.

The surface considered apart from the substance is called a *geometrical surface*.

10. A Plane is a *geometrical surface* such that, if any two points in it be selected at random, the straight line joining them will lie wholly in that surface.

11. A Curved Surface is a *geometrical surface* no portion of which is a plane.

12. A physical solid is the material composing it, and which we perceive through the medium of the senses; while the *geometrical solid* is the *space*, simply, which the physical solid occupies.

13. A Geometrical Figure is the term applied to combinations of points, lines, and surfaces, when reference is had to their form or outline simply.