

**MATHEMATICAL QUESTIONS, WITH  
THEIR SOLUTIONS, FROM THE  
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WITH MANY PAPERS AND SOLUTIONS  
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**W. J. C. MILLER**

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### Solved Questions.

3108. (J. J. Walker, M.A.)—Through B, C, angles of a spherical triangle ABC, draw arcs perpendicular to AB, AC, respectively, and meeting in D. Let AE, AF be arcs equally inclined to AB, AC, and meeting the former the base BC in E, the latter a perpendicular arc through D in F; prove that
- $$\tan AE \cdot \tan AF = \tan \delta \cdot \tan \epsilon,$$
- $$\tan^2 AD = \frac{\tan^2 \delta + \tan^2 \epsilon - 2 \tan \delta \cdot \tan \epsilon \cos A}{\sin^2 A} \dots\dots 38$$
3220. (J. J. Walker, M.A.)—The theorem in Question 3122 may be generalized by supposing that PO bears any constant ratio ( $k$ ) to the conjugate semi-diameter.
1. Prove that the circle passing through LMN, the point P referred to the axes of the ellipse being  $(x', y')$ , is
 
$$x^2 + y^2 - \frac{\delta(\delta - ka)}{a^2} x'x - \frac{a(a - kb)}{b^2} y'y - (a^2 + b^2 - kab) = 0.$$
  2. Verify that this circle passes through the point on the ellipse diametrically opposite to P; and find its envelope as P moves round the ellipse.
  3. If the normals at L, M, N meet the ellipse which is the locus of O again in L', M', N', prove that LL', MM', NN' are to the semi-diameters parallel to the tangents at L, M, N respectively as  $k$  to 1..... 102



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$\int_0^{\infty} \frac{dx}{e^{ux^2} - e^{-ux^2}} = -\text{Ei}(-u) - \text{Ei}(-3u) - \text{Ei}(-5u) - \dots,$	
Ei denoting the exponential-integral. As a particular case, putting $u=1$ ,	
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$OO' = AO \left( \frac{1}{OP_2} - \frac{1}{OP_1} \right) \div \left( \frac{1}{OP_2} - \frac{1}{OP_1} \right),$	
AO being less than one-half the length of the axis .....	19
3541. (The Editor.)—A piece of slender uniform wire, of indefinite length, every point of which attracts with a force varying according to the inverse square of the distance, is bent into the form of the catacaustic of the parabola for rays perpendicular to the axis, (or, what is the same curve, the first negative focal pedal of the parabola); and A <sub>1</sub> , A <sub>2</sub> , A <sub>3</sub> are the several attractions on a particle at the focus, of the whole curve, of the loop of it, and of the part which exerts the greatest attraction towards the vertex; prove (1) that A <sub>1</sub> : A <sub>2</sub> : A <sub>3</sub> = 8 : 6 $\sqrt{3}$ : 19, and (2) that an arc of the curve extending from the vertex to an angular distance from the focus of 23° 44' 2" on each side of the axis would exert the same attraction as the whole curve on a particle at the focus .....	17
3542. (G. O'Hanlon.)—Find the normal which cuts off the least curve from a conic .....	29
3546. (R. Tucker, M.A.)—The angles of a triangle are taken as the eccentric angles of three points P, Q, R on an ellipse; find the envelope of the locus of the centres of the circles about PQR (P being supposed at first fixed, and then to vary its position). Find also the locus of the centres of the circles which the circles touch when a point is fixed .....	68