MATHEMATICAL QUESTIONS, WITH THEIR SOLUTIONS, FROM THE "EDUCATIONAL TIMES", WITH MANY ADDITIONAL SOLUTIONS NOT PUBLISHED IN THE "EDUCATIONAL TIMES", VOL. II, FROM JULY TO DECEMBER, 1864

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# W. J. MILLER

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BDITED BY

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