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WITH THEIR

SOLUTIONS,

FROM THE "EDUCATIONAL TIMES,"

WITH MANY

Papers and Solutions not published in the "Educational Times."

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Solved Questions.

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1843. Three points being taken at random within a circle, find the chance that the circle drawn through them will lie wholly within the given circle..... 17, 95

2534. It is a well known property in Geometry of Two Dimensions, that, when a system of conics have double contact, a variable chord of any one of them cut in a constant anharmonic ratio by any other of them, (α) is cut in constant anharmonic ratios by them all, (δ) touches the same one of them in every position, and (ϵ) determines on every one of them two homographic systems of points, of which the two common points and lines of contact are double points and lines.

Show, analogously in Geometry of Three Dimensions, that when a system of quadrics have quadruple contact, (that is, when they pass through the four sides of a common quadrilateral, real or imaginary in space,) a variable chord of any one of them cut in constant anharmonic ratios by any two of them, (α) is cut in constant anharmonic ratios by them all,

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(b) touches the same two of them in every position, and (c) determines on every one of them two homographic systems of points, of which the four common points and planes of contact are double points and planes. 54

2570. Find the number of ways in which the first 9 digits may be arranged so as to make up 99. 104

2700. Three equal coins at piled at random on a horizontal plane; required the probability that the pile will stand. 34, 111

2709. A series of curves being determined by the elimination of θ between

$$X = e^{m\theta}(m^2\cos 2\theta + 2m\sin 2\theta + m^2 + 4),\quad Y = e^{m\theta}(m^2\sin 2\theta - 2m\cos 2\theta),$$

where $$X = \frac{2x}{c}m(m^2+4) + 2(m^2+2),$$

and $$Y = \frac{2y}{c}m(m^2+4) - 2m;$$

show that when $m = 0$ the particular curve will be a cycloid... 64

2711. To resolve any given number into three rational cubes. 63

2822. A raffling match is composed of 5 persons, each throwing 3 times with 7 pennies, the one turning up the greatest number of heads to be winner. The third player having turned up 15 heads, it is required to determine his chance of winning. 29

2836. Two conics osculate at O and intersect at P; if any straight line be drawn through P, the locus of the intersection of tangents, drawn to the conics at the points where this line meets them, is a conic touching the former at O, and also touching them again, and the curvature at O of this locus is three-fourths of the curvature of either of the given conics. 46

2837. If the numbers a, b, c be the sides of a triangle, prove that

$$\tfrac{1}{3}(a+b+c)(a^2+b^2+c^2) > a^3+b^3+c^3+2abc. \ldots\ldots 30$$

2845. Find a complete solution of the equation in second differences

$$u_x = u_{x-1} + (x-1)(x-2)u_{x-2}. \ldots\ldots 50$$

2847. Prove that $e^{kD^2}.e^{-kx^2} = (1+4k^2)^{-\frac{1}{2}}e^{-\frac{kx^2}{1+4k^2}}$, where $D = \frac{d}{dx}$. 51

2851. From a given point a straight line is drawn to meet the tangent to a given conic, so that the two straight lines may be conjugate with respect to a second given conic. The locus of the point of intersection is a quartic curve, whose equation may be expressed in the form $U^2+V^2+W^2=0$, where U, V, W are certain conics. 68

2854. Solve the equation $x^x = a$, and find the value of x when $a = 300$. 40

2856. If two secants SP, SP′ turning round a fixed point S as pole, so as to intercept on a given circle PP′Q′ an arc PP′, whose middle A is a given point, meet the circle again in Q and Q′; prove that the line QQ′ turns round a pole 48

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2882. 1. ABC is any equilateral triangle, formed by three arcs of equal circles: if AC, BC be produced to meet in C′, prove that AC′ = BC′ = 60°.

2. Any three equal circles ABB′A′, ACC′A′, BCC′B′ form by their intersections the circular triangles ABC, A′B′C′ (C, C′ being within the circle ABB′A′); prove that the arcs AC + BC − AB = A′C′ + B′C′ − A′B′. 41

2905. Mr. Punch's renown
In London town
Brought up in dozens
His country cousins
Twenty-eight ladies, pretty and shy,
Twenty-one gentlemen, six feet high.

Quoth he, "I invite
Four couples a night,
A belle with a beau,
Whenever you choose;
If only, you know,
Just now, in the season,
You have the discretion
This rule to use,—
That never a pair
Of you all shall share
Together twice my evening fare."

Then smiled and bowed
The happy crowd,
In full content;
And beau with belle,
The hungry sinners,
In eights they went,
And polished off well
Just twenty-one dinners.

They were loth to leave when all was o'er,
And the rule forbade an octave more.
Then went Mr. Punch on,
"I bid you to luncheon,
A beau with a belle,
In couples three;
But look to it well
I never see
Two meet who have met at table with me.

The joy was loud
Of the happy crowd;
And twenty-eight noons,
In sixes merry,
They plied his spoons
And drank his sherry.

Then, to the fair who alone, as yet,
In his banquet hall had never met,
He said, "My dears, (it can't be improper,) arrange to go with me all to the Opera;
Come only in flocks of pairs never able,
To meet in my box or meet at my table."

Then for eight nights,
Oh, all in their best
So charmingly drest,
Came ravishing sights,
In bevies of seven;
And, girt and caress'd
By the dear delights
Mr. Punch was blest
With peeps at heaven.

Have you the skill
The lists to fill,
And of forty-nine
All pairs combine? 78

2911. P is a fixed point on the outer of two confocal ellipses; and QPR is the tangent at P. Two variable parallel tangents to the same conic meet the fixed tangent in Q and R; and from Q and R tangents are drawn to the inner ellipse intersecting in O. Show that the locus of O is a circle, having its centre on the normal at P. 31

2918. From any point in a given line tangents are drawn to a cissoid, and the circle described through the points of contact: prove that the envelope of the radical axis of this and the generating circle is in general a conic passing through the cusp, which becomes an ellipse having its minor axis coincident with the tangent at cusp when the given line passes through a certain point in that tangent; one of the three conics with an axis similarly directed when the given line is perpendicular to the tangent at cusp; and the generating circle itself when both the above conditions are fulfilled. 87

2939. Chords of an ellipse are drawn subtending a right angle at a fixed point O, and O′ is the second focus of the envelope of