SYLLABUS OF PLANE' GEOMETRY (CORRESPONDING TO EUCLID, BOOK I-VI)

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CORA L. WILLIAMS

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(Corresponding to Euclid, Book I - VI)

PREPARED AS AN INTRODUCTION
TO ABSOLUTE GEOMETRY BY
CORA L. WILLIAMS, M. S.

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PREFACE

THE logical order in geometry has held the attention of mathematicians for more than two thousand years, but no essential progress was made towards its solution, after the time of Euclid, until the nineteenth century witnessed the creation and development of the non-Euclidean geometry. A radically new departure was taken when Lobatschewzky and Bolyai demonstrated that a consistent geometry could be constructed under an assumption which contradicts the parallel axiom. This took place before the year 1830, yet no serious attempt was made to reclassify the subject matter of geometry until the close of the century. The present essay, prepared as a thesis for the master's degree in the year 1898, attempts a classification based upon the three hypotheses which may be made with reference to the existence of parallels. It makes no use of the other independent postulates of geometry enunciated for the first time by Hilbert in the year 1899. A more minute classification than the one here presented is therefore possible, but what order is best adapted to the systematic unfolding of geometry for the beginner has not yet been determined. Meanwhile we have at hand in this essay a working plan which may be used to good effect, before the study of geometry is abandoned by the pupil, in exhibiting its logical structure. A course of geometry without some such syllabus is like an arch without a keystone; it is certain to fall into fragments.

IRVING STRINGHAM.

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Syllabus of Plane Geometry

DEFINITIONS

Def. 1. Any physical object takes up room. The room occupied by an object, considered apart from the object, is called a solid.

Def. 2. That which separates a solid from any part of the room surrounding it, but itself no solid, is called a surface.

Def. 3. That which separates one part of a surface from an adjacent part is called a line.

Def. 4. That which separates one part of a line from an adjacent part is called a point.

A point is not divisible; it has position only.

The intersection of two lines is a point. The intersection of a surface and a line is a point. The intersection of two surfaces is a line.

Def. 5. Points, lines, surfaces, or solids, or any combination of them, are called figures.

Def. 6. Any assumption concerning the relations of figures to one another is called a postulate.*

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* See Helmholtz: The Origin and Significance of Geometrical Axioms, Popular Scientific Lectures.

Def. 7. When a series of figures existing in accordance with certain given postulates form only other figures belonging to the same series, they are said to constitute a geometry.

A geometry can yield no relation between its figures that is not already contained in its postulates, but the statement of certain relations is essential to the complete determination of its figures. Such statements, capable of being established from previous assumptions, are called theorems. These previous assumptions may, themselves, be theorems or postulates.

It is evident that only those theorems are fundamental to a geometry which are required to establish its integrity. The other theorems, practically unlimited in number, express relations between figures in general.

Def. 8. The figure which contains the figures, as they exist in accordance with the postulates of a geometry, is called the space, or space form of that geometry.

The effect of introducing into a geometry a new postulate is to define more completely the character of its space-form.

Any postulate may be introduced into a

geometry which does not destroy the integrity of the geometry.

Def. 9. The space-form of a geometry is completely determined when no postulate can be added to the geometry which is not an equivalent of a postulate already included in it.

Def. 10. When the space-form of a geometry is completely determined, the geometry may be said to be logically complete.

Def. 11. The geometries which are logically complete for the same series of figures together constitute what is called the absolute geometry of those figures.