

**A TREATISE ON ANALYTICAL  
GEOMETRY; WITH APPLICATIONS  
TO LINES AND SURFACES OF  
THE FIRST AND SECOND ORDERS**

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A Treatise on Analytical Geometry; With Applications to Lines and Surfaces of the First and Second Orders by William G. Peck

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**WILLIAM G. PECK**

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A TREATISE  
ON  
ANALYTICAL GEOMETRY,

WITH

APPLICATIONS TO LINES AND SURFACES OF THE  
FIRST AND SECOND ORDERS.

BY

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## P R E F A C E .

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THE following Treatise on Analytical Geometry has been prepared for the use of the mathematical department in Columbia College and the School of Mines. Many excellent works on this branch of mathematics have been published, but the larger ones have proved too voluminous to be studied in the time allotted to the subject in colleges and schools of science, and the smaller ones are not sufficiently comprehensive to satisfy the growing wants of scientific education. The object of the present compilation is to deduce all the essential principles usually developed in the larger works, within the limits occupied by those of the smaller class.

The general plan of the work does not differ essentially from that adopted by the earlier writers on the subject, but in its execution, many changes have been introduced. The definitions have been revised, the explanations have been simplified, the demonstrations have been abbreviated, and every

branch has been illustrated by problems, intended to test the student's knowledge of the principles demonstrated. The method of treating tangents, normals, subtangents, and subnormals, has been much abridged, and, it is believed, correspondingly improved. Much care has been bestowed on the discussion of the general equation of the second degree, particularly with respect to the methods of testing the nature of the different-curves that are represented by it.

The wood-cuts used in illustration have been kindly loaned by Prof. Davies. In thanking him for this act of courtesy, the writer begs leave also to acknowledge his indebtedness to him for other valuable aid. The continuous use of his text-books for more than a quarter of a century has not failed to leave an impress on the following pages.

COLUMBIA COLLEGE, }  
June 1, 1873. }

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