

**ELEMENTS OF THE
DIFFERENTIAL AND INTEGRAL
CALCULUS WITH EXAMPLES
AND APPLICATIONS**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649571321

Elements of the Differential and Integral Calculus with Examples and Applications by James M. Taylor

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

JAMES M. TAYLOR

**ELEMENTS OF THE
DIFFERENTIAL AND INTEGRAL
CALCULUS WITH EXAMPLES
AND APPLICATIONS**

ELEMENTS
OF THE
DIFFERENTIAL AND INTEGRAL
CALCULUS,
WITH
EXAMPLES AND APPLICATIONS.

BY
JAMES M. TAYLOR,
PROFESSOR OF MATHEMATICS, MADISON UNIVERSITY.



BOSTON:
PUBLISHED BY GINN & COMPANY.
1886.

Educ 17 188.86.810

✓
HARVARD COLLEGE LIBRARY
Mrs. Albert Sauvour,
Cambridge,
July 20, 1959

Entered according to Act of Congress, in the year 1884, by
JAMES M. TAYLOR,
in the Office of the Librarian of Congress, at Washington.

PREFACE.

THE object of the following treatise is to present simply and concisely the fundamental problems of the Calculus, their solution, and more common applications.

Since variables are its characteristic quantities, the first fundamental problem of the Calculus is, *To find the ratio of the rates of change of related variables.* To enable the learner most clearly to comprehend this problem, the author has employed the conception of rates, which affords finite differentials and the simplest demonstration of many principles. The problem of Differentiation having been clearly presented, a general method of its solution is obtained by the use of limits. This order of development avoids the use of the indeterminate form $\frac{0}{0}$, and secures all the advantages of the differential notation. Many principles are proved, both by the method of rates and that of limits, and thus each is made to throw light upon the other.

In a final chapter, the method of infinitesimals is briefly presented; its underlying principles having been previously established.

The chapter on Differentiation is followed by one on Integration; and in each, as throughout the work, there

are numerous practical problems in Geometry and Mechanics, which serve to exhibit the power and use of the science, and to excite and keep alive the interest of the student.

In writing this treatise, the works of the best American, English, and French authors have been consulted; and from these sources the most of the examples and problems have been obtained.

The author is indebted to Professors J. E. OLIVER and J. MCMAHON of Cornell University, and Professor O. ROOT, Jr., of Hamilton College, for valuable suggestions; and to Messrs. J. S. CUSHING & Co. for the typographical excellence of the book.

J. M. TAYLOR.

HAMILTON, N. Y.,
Nov., 1884.

CONTENTS.

CHAPTER I.

INTRODUCTION.

Section.	Page.
1. Definition of <i>variable</i> and <i>constant</i>	1
2. Definition of <i>function</i> and <i>independent variable</i>	1
3. Classification of functions	2
4. Definition of <i>continuous variable</i> and <i>continuous function</i>	2
5. Definition of the <i>limit</i> of a variable	3
6. <i>Limits</i> of equal variables	3
7. Limit of the <i>product</i> of a constant and a variable	4
8. Limit of the <i>product</i> of two or more variables	4
9. Limit of the <i>quotient</i> of two variables	4
10. Limit of the <i>sum</i> of two or more variables	4
11. Definition of <i>uniform</i> change	5
12. Definition of <i>increment</i>	5
13. Definition of <i>differential</i>	5
14. Illustrations of differentials	6
15. Definition of <i>inclination</i> , <i>slope</i> , and <i>tangent</i>	7
16. Geometric signification of $\frac{dy}{dx}$	7
17. Limit of the ratio of the increments of <i>y</i> and <i>x</i>	8

CHAPTER II.

DIFFERENTIATION.

18. Definition of <i>differentiation</i> . Differentiation of ax^2	10
--	----

Algebraic Functions.

19. Differential of the <i>product</i> of a constant and variable	10
20. Differential of a <i>constant</i>	11
21. Differential of the <i>sum</i> of two or more variables	11

Section.	Page.
22. Differential of the <i>product</i> of two variables	12
23. Differential of the <i>product</i> of several variables	13
24. Differential of a <i>fraction</i>	14
25. Differential of a variable with a <i>constant exponent</i>	14
26. General symbol for the differential of $f(x)$. <i>Examples</i>	15
27. Definition of an <i>increasing</i> and a <i>decreasing</i> function	17
28. Definition of <i>derivative</i>	17
29. <i>Measure</i> of rate of change	18
30. Signification of $\frac{dy}{dt}$	18
31. Signification of $f'(x)$ or $\frac{dy}{dx}$	18
32. Limit of the ratio of Δy to Δx . <i>Applications</i>	19
33. Definition of <i>velocity</i> and <i>acceleration</i> . <i>Examples</i>	23
<i>Logarithmic and Exponential Functions.</i>	
34. Differential of a <i>logarithmic function</i>	24
35. The greater the base, the smaller the modulus	25
36. <i>Naperian system</i>	25
37. Differential of e^x	26
38. Differential of y^a	26
39. <i>Logarithmic differentiation</i> . <i>Examples</i>	26
<i>Trigonometric Functions.</i>	
40. Definition of the <i>unit</i> of angular measure	29
41. Differential of $\sin x$ and $\cos x$	29
42. Differential of $\tan x$	30
43. Differential of $\cot x$	30
44. Differential of $\sec x$	30
45. Differential of $\operatorname{cosec} x$	31
46. Differential of $\operatorname{vers} x$	31
47. Differential of $\operatorname{covers} x$	31
48. Limit of the ratio of an arc to its chord	31
49. Differentiation of $\sin x$ by the method of limits. <i>Examples</i>	32
<i>Anti-Trigonometric Functions.</i>	
50. Differential of $\sin^{-1}x$	35
51. Differential of $\cos^{-1}x$	35
52. Differential of $\tan^{-1}x$	35
53. Differential of $\cot^{-1}x$	36
54. Differential of $\sec^{-1}x$	36

CONTENTS.

vii

Section.	Page.
55. Differential of $\operatorname{cosec}^{-1}x$	36
56. Differential of $\operatorname{vers}^{-1}x$	36
57. Differential of $\operatorname{covers}^{-1}x$. Examples	36
Miscellaneous examples	39

CHAPTER III.

INTEGRATION.

58. Definition of <i>integral</i> and <i>integration</i> . Sign of integration . . .	43
59. Elementary principles	43
60. Fundamental formulas	44
61. Statement of formulas 1 and 2. Examples	46
62. Auxiliary formulas. Examples	50
63. Trigonometric differentials. Examples	56
64. <i>Definite integrals</i> . Examples	58

Applications to Geometry and Mechanics.

65. Rectification of curves. Examples	60
66. Areas of plane curves. Examples	61
67. Graphical representation of any integral	63
68. Areas of surfaces of revolution. Examples	63
69. Volumes of solids of revolution. Examples	65
70. Fundamental formulas of mechanics. Examples	66
1. Formulas for uniformly accelerated motion.	
2. Motion down an inclined plane.	
3. Motion down a chord of a vertical circle.	
4. Values of v and s when a varies directly as t .	
5. Geometrical representation of the <i>time, velocity, distance,</i> <i>and acceleration.</i>	
6. Path of a projectile.	
7. Path, velocity, and acceleration of a body whose velocity in each of two directions is given.	

CHAPTER IV.

SUCCESSIVE DIFFERENTIATION.

71. Successive derivatives	71
72. Signification of $f''(x), f'''(x), f^n(x)$. Examples	71
73. Successive differentials	73
74. Relations between successive differentials and derivatives. Examples	73