A TREATISE ON THE THEORY AND SOLUTION OF ALGEBRAICAL EQUATIONS

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JOHN MACNIE

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BY

JOHN MACNIE, M.A.

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1876.

ERRATA.

Page 2, line 28, for $\frac{1}{2}$ read $\frac{1}{2C_2}$.

" 8, " 25, for *root* read *real root*.

" 19, " 2, for *square* read *square root*.

" 19, " 26, for $-\sqrt{-1}$ read $+\sqrt{-1}$.

" 28, " 2, for $X +$ read X.

" 43, " 3, for X^4 read X^2.

" 61, " 11, for *com. div.* read *greatest com. div.*

" 82, " 20, dele Tischendorf.

" 85, " 7, for $(yz+yu+zu)$ read $(yz+yu+zu)^2$

" 85, " 10, for $(y^2+z^2+u^2)$ read $(y^2+z^2+u^2)^2$

" 129, " 24, for *pairs* read *groups*.

" 147, " 15, for $(-a-\beta)$ read $-(a-\beta.)$

PREFACE.

THE following treatise is designed to present, as succinctly as is consistent with a due presentation of the subject, the general theory of algebraical equations, while special attention is given to the analysis and solution of equations with numerical coefficients.

The work may be regarded as a complement to the more advanced treatises on algebra, in which the general theory of equations, if discussed at all, is necessarily compressed into a space altogether inadequate to a satisfactory exposition of this important branch of mathematical study. In order to preserve this character of the work as a sequel to ordinary algebra, algebraical methods have been carefully adhered to throughout, except in a few articles, in which trigonometrical expressions have unavoidably been introduced. The treatise may therefore be easily read by those who have passed through the usual course in algebra.

The work had its first inception in some vain attempts at so modifying Sturm's Method as to lessen the great labor attending an analysis by that method of most equations of above the fourth degree. Becoming convinced that, from the nature of the case, no such modification is possible that does not impair the certainty which constitutes the chief excellency of the method, the author turned his attention to an investigation of the possibility of a satisfactory analysis by means of Fourier's Theorem. This he soon saw could be effected if, by any means, the presence of imaginary roots in a given interval could be readily ascertained. The results of this investigation are given (Chap. X) in a method of analysis based upon Fourier's Theorem, a method that possesses the merit of at least great facility as compared with that of Sturm.

The method involves, however, an extension of the application of Horner's Method, especially a generalization of the principle of trial divisors, not given in the great majority of treatises upon equations. This first suggested the idea of writing a short account of Horner's Method in which its capabilities should be fully exhibited, with the method of analysis based on that method in conjunction with Fourier's Theorem. A natural extension of that idea led to the present treatise, the first upon this subject that, as far as the author has been able to ascertain, has been published in the United States.

The treatise, though kept within its present dimensions by the exclusion of much that had been prepared, will be found to contain all the propositions generally given in an elementary treatise upon this subject, with a few exceptions. Thus Newton's method of approximation, and that of Lagrange, have been excluded, as being entirely superseded by that of Horner, to which, even in the most favorable cases, they are inferior in symmetry, compactness, and facility. The theory of determinants has not been introduced, on the ground that a suitable account would require to itself a volume of the dimensions of the present treatise. The student desirous of information upon the foregoing and other omitted topics may consult Todhunter's excellent treatise, the best, upon the whole, with which the author is acquainted.

On account of the interest naturally attaching to the subject, the algebraical solution of equations has (Chap. VII) been treated of at some length. In Art. 121, 133–146, are given some results that may be found of interest, and which, it is believed, are now published for the first time.

A chapter (VIII) has been devoted to Sturm's Theorem, thus enabling the student to form for himself a clear estimate of the peculiar excellencies and disadvantages of the method of analysis based upon that theorem, and to institute a fair comparison between it and the method explained in Chap. X.

A short chapter on cubic equations has been inserted, in which a method of procedure is given that relieves the solution of the cubic from much of its tentative character, and reduces the arithmetical labor to a minimum.

To render the treatise more convenient for the work of the class-room, the subject-matter has been thrown, as far as possible, into the form of propositions with their dependent corollaries. The number of exercises given will, it is hoped, be found sufficient in number to illustrate every part of the subject and not so difficult as to needlessly consume time.

JOHN MACNIE.

NEWBURGH, *August*, 1875.

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FUNDAMENTAL PROPERTIES OF EQUATIONS.

CHAPTER II.

IMAGINARY EXPRESSIONS. CAUCHY'S THEOREM.