TABLES FOR THE FORMATION OF LOGARITHMS & ANTI-LOGARITHMS TO TWENTY-FOUR OR ANY LESS NUMBER OF PLACES; WITH EXPLANATORY INTRODUCTION AND HISTORICAL PREFACE

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Tables for the Formation of Logarithms & Anti-Logarithms to Twenty-Four or Any Less Number of Places; With Explanatory Introduction and Historical Preface by Peter Gray

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BY

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PREFACE.

I PROPOSE to give here a brief account of the origination of the methods for the formation of logarithms and anti-logarithms which are developed in the succeeding pages.

A systematic and practical method for the formation of logarithms will, probably of necessity, consist in the resolution of the number whose logarithm is required into factors, and the addition of the logarithms of these factors, taken from a table previously prepared; the sum being the logarithm required. Also, the formation of an anti-logarithm will be effected by the decomposition of a given logarithm into a series of logarithms taken from a table, and the multiplication of the corresponding numbers, the product being the required anti-logarithm. The extent of the data is minimized if, which is not necessarily the case, the same table is used in both the direct and the inverse processes; and both processes will depend more or less on the form of the factors whose logarithms are tabulated.

The earliest systematic logarithmic method with which I am acquainted is Mr. Manning's, of which an account is given in the *Philosophical Transactions* for 1806. Mr. Manning's paper is reprinted, ("nearly as it stands," Mr. Young says,) in Young's "*Elementary Essay on the Computation of Logarithms*."* Mr. Manning first reduces the number to be dealt with, in the manner described on pp. [3] and [32] of the following Introduction, to the form $1+N_1$, where N_1 is a decimal fraction; and he then multiplies the number thus modified *down* to unity, by the employment of a succession of factors, $1-\cdot1$, $1-\cdot01$, $1-\cdot001$, &c., (the general form being $1-\cdot1^r$, where r takes the values 1, 2, 3, &c., successively,) till the decimal portion of $1+N_1$ is exhausted. It is obvious that the product of the factors by which this exhaustion is effected will be the reciprocal of $1+N_1$; and hence that the sum of their co-logarithms will be the logarithm of $1+N_1$. It is the co-

* Second Edition, London, 1835, pp. 67 to 79.

logarithms, with the first nine multiples of each, which form the data in this method.

Mr. Manning's method is remarkably simple, but it is also exceedingly tedious. Multiplication by a factor of the form $1-\cdot1^r$, is effected by subtracting from the multiplicand the multiplicand itself, removed r places to the right; and the resolving process thus consists of series of subtractions of the simplest character, which moreover do not need to be formally continued after the exhaustion of the first half of the decimal places in N_1. Still, with this abatement, the number of subtractions requisite remains so numerous as to render the process exceedingly irksome. In an example of it now before me, the formation of log π to sixteen places, the number of subtractions is thirty-eight, occupying seventy-six lines, and the number of tabular entries is twenty-one.

The next step, constituting a great advance, was made by Mr. Thomas Weddle, subsequently Professor in the Royal Military College, Sandhurst. In *The Mathematician* for November, 1845, Mr. Weddle gives an account of both a logarithmic and an anti-logarithmic method, "discovered," he says, "nearly seven years ago." The first of these methods he describes as a modification of Mr. Manning's. Mr. Manning's factors are of the form $1-\cdot1^r n$, when n is always unity; Mr. Weddle's are of the same form, but in them n takes all values from 1 to 9; the effect of which is, that at the cost of a few multiplicators by factors of a single digit, the number of subtractions is reduced by about four-fifths. In the example already referred to, involving by Mr. Manning's process thirty-eight subtractions, six only are needed when it is worked by Mr. Weddle's. The number of tabular entries is about the same in both processes.

Mr. Manning did not attempt to apply his data to the converse problem. Mr. Weddle, as intimated, made this application of his data. His tabulated logarithms, (extending to sixteen places,) being those of the reciprocals of the factors, he decomposes the complement of the given logarithm, by subtraction, into a series of values taken from his table; and multiplication of the corresponding factors gives the required number.

The next step, as I suppose, was taken by myself. It consisted in the construction of a new table, to twelve places, in which the factors were of the form $1-(\cdot01)^r n$, permitting n consequently to take any value of two figures, that is, from 1 to 99. The effect of this was, in both the direct and the inverse processes, a reduction in the number of tabular entries of nearly one-half, and in the number of figures of about two-fifths.

In my table, which, with the requisite description and examples, appeared in the *Mechanics' Magazine* for October and November, 1846, the data were for the first time arranged in columns, corresponding to successive values of r; while the values of n occupy the argument column.

It will be convenient, in indicating the extent of any specified table, to designate it as a one-figure, a two-figure, a three-figure, &c. table, according as its argument consists of one, two, three, &c. figures, respectively. My table, just referred to, was a two-figure table, and Mr. Weddle's was a one-figure table.

The next publication was a paper by Mr. Hearn, of the Royal Military College, Sandhurst, which appeared in the *Mathematician* for March, 1847. It was entitled "*Practical Method of forming Logarithms and Anti-Logarithms, independently of Extensive Tables;*" and it contained two one-figure tables, to ten places, of which the first was intended for the formation of logarithms, and the second for the formation of anti-logarithms. The first of these was similar to Mr. Weddle's table, and the manner of using it was the same. It therefore needs no further remark here. In the second table the factors, instead of $1-(\cdot1)^r n$, were of the form $1+(\cdot1)^r n$; so that in its application to the purpose for which it was intended, the subtractions previously requisite were replaced by additions, thus materially improving the anti-logarithmic operation.

I saw that this would be still further improved by the adaptation to it of a two-figure table. I accordingly constructed such a table to twelve places, and I found that in its use my expectations were fully realized. The number of tabular entries was reduced one-half, and the number of additions in a still higher ratio.

My paper, descriptive of this anti-logarithmic method, and accompanied by the new table, appeared in the *Mechanics' Magazine* for February 12, 1848. Soon after the preparation of this paper, I discovered a method of applying a table of Mr. Hearn's form to the construction of logarithms; and I described this method, with an example, in a paper which appeared in the same journal a fortnight later, in the number for February 26.

The result of my extension of the two tables had proved so satisfactory, as regards the abbreviation and improvement in other respects of the processes, that I now resolved on the construction of a three-figure table, in which the data should extend to twenty-four places, and which would therefore be available for formations to the number of places named. I adopted for the extended table the form proposed by Mr. Hearn, as possessing on the whole the greatest facilities for both the direct and the inverse operations.

It is the table resulting from this extension that forms the basis of the present Work. The methods employed in the construction and verification of it are briefly described in Section IV. of the Introduction. The labour attending these operations, (in which I had no assistance,) was no doubt very considerable; but, from the evidence already acquired of the power and utility of the table, I am satisfied that the labour was not ill bestowed.

The table was completed upwards of twenty years ago. Knowing it was not of a character likely soon to repay the cost of printing, I suffered it to lie by me in manuscript, (using it occasionally for my own purposes,) for a number of years. At length I made an abridgment of it, adapted for formations of twelve places, which, with the requisite explanatory matter, I communicated to the *Assurance Magazine*, in the twelfth volume of which my papers appeared.* They were subsequently collected and issued in a separate form, in 1865, by Messrs. Layton. This tract, after some years, came under the notice of a gentleman with whom I had no previous acquaintance, Thomas Warner, Esq., F.R.A.S., of 47, Sussex Square, Brighton. Mr. Warner opened a correspondence with me, in the course of which he was pleased to speak very favourably of the probable utility of the large table, and he offered me a most handsome contribution towards the expense of putting it in print. Having mentioned this circumstance to two gentlemen interested in such matters, they each offered, quite spontaneously, a liberal contribution, in supplement of Mr. Warner's, towards the same object. I have pleasure in naming the gentlemen who gave this gratifying proof of their appreciation of the value of the table. They were, J. W. L. Glaisher, Esq., F.R.S., of Trinity College, Cambridge, and H. D. Hoskold, Esq., Civil and Mining Engineer, of Dean Forest, Gloucestershire.

Encouraged by the approval of such competent judges, I put the table to press. During the preparation of the introductory matter, I have had many suggestions, by which I have profited, from Mr. Glaisher, Mr. Warner, and my friend Major-Gen. Hannyngton, to all of whom I desire here to record my thanks. To Mr. Glaisher, in particular, for the kindly interest he has taken in the progress of my Work, and the encouragement thus afforded me, my special thanks are due.

It may be fitting that, ere I close, I should advert briefly to other

* I think it needful to mention that, in the papers referred to, I explained and exemplified the use of both Weddle's and Hearn's one-figure tables, and of my own two-figure tables.

methods that have been proposed for the formation of logarithms, akin to those I have described, and to other publications of the same or similar methods.

The first in order of date with which I am acquainted are two methods suggested by the late Mr. William Orchard, in a letter which appeared in the *Mechanics' Magazine* for Feb. 26, 1848, the same number which, as already intimated, contained the first example of my present logarithmic method. In his first method, Mr. Orchard, availing himself of my extended table of Hearn's form, dealing with the number whose logarithm is required as a decimal fraction, multiplies it *up* to unity by the use of the tabular factors. The sum of the logarithms of the factors employed is the co-logarithm of the given number. The operation in this method is a very compact one.

In Mr. Orchard's second method he proposed to multiply *up* the number as before, by a series of factors of the form $(1{\cdot}1)^n$, $(1{\cdot}01)^n$, $(1{\cdot}001)^n$, &c., n having in each case a *suitable* value, never exceeding 9; and he used the binomial factors in the multiplications. As the method was only suggested, Mr. Orchard gave no table for its application.

The next in order is Mr. Oliver Byrne, whose method is described in a work published in 1849, entitled, "*Practical, Short and Direct Method of Calculating the Logarithm of any Given Number, and the Number corresponding to any Given Logarithm.*" Mr. Byrne informs us in his Introduction that he discovered his method "about twenty years ago."

Mr. Byrne having, by an ingenious application of Lagrange's Theorem, provided himself with a series of ten numbers, ranging from 1 to 10, the *figures* of which are the same as those of their logarithms, respectively, generally multiplies the given number *up* to the next greater of these tabulated numbers, by the requisite factors of the form employed in Orchard's second method; effecting the multiplications, also as in Orchard's method, by aid of the binomial co-efficients. The sum of the logarithms of the factors, subtracted from the number employed (or its logarithm) is the logarithm required. The method, it thus appears, is virtually Orchard's, the main distinction being that, in the last named, the final subtraction is the formation of an arithmetical complement.

I refer next to Gen. Shortrede's *Logarithmic Tables*, published in 1849. Gen. Shortrede being home on furlough, and occupied with the issue of his tables, my papers in the *Mechanics' Magazine* came under his notice, as also Weddle's and Hearn's papers in the *Mathematician*. He computed extended tables of the forms used by those gentlemen, and introduced them into his volume as follows:—first, a two-figure