# PLANE GEOMETRY

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Plane Geometry by G. A. Wentworth



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BY

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# PREFACE.

Most persons do not possess, and do not easily acquire, the power of abstraction requisite for apprehending geometrical conceptions, and for keeping in mind the successive steps of a continuous argument. Hence, with a very large proportion of beginners in Geometry, it depends mainly upon the *form* in which the subject is presented whether they pursue the study with indifference, not to say aversion, or with increasing interest and pleasure

Great care, therefore, has been taken to make the pages attractive. The figures have been carefully drawn and placed in the middle of the page, so that they fall directly under the eye in immediate connection with the text; and in no case is it necessary to turn the page in reading a demonstration. Full, long-dashed, and short-dashed lines of the figures indicate given, resulting, and auxiliary lines, respectively. Bold-faced, italic, and roman type has been skilfully used to distinguish the hypothesis, the conclusion to be proved, and the proof.

As a further concession to the beginner, the reason for each statement in the early proofs is printed in small italics, immediately following the statement. This prevents the necessity of interrupting the logical train of thought by turning to a previous section, and compels the learner to become familiar with a large number of geometrical truths by constantly seeing and repeating them. This help is gradually discarded, and the pupil is left to depend upon the knowledge already acquired, or to find the reason for a step by turning to the given reference.

It must not be inferred, because this is not a geometry of interrogation points, that the author has lost sight of the real object of the study. The training to be obtained from carefully following the logical steps of a complete proof has been provided for by the Propositions of the

Geometry, and the development of the power to grasp and prove new truths has been provided for by original exercises. The chief value of any Geometry consists in the happy combination of these two kinds of training. The exercises have been arranged according to the test of experience, and are so abundant that it is not expected that any one class will work them all out. The methods of attacking and proving original theorems are fully explained in the first Book, and illustrated by sufficient examples; and the methods of attacking and solving original problems are explained in the second Book, and illustrated by examples worked out in full. None but the very simplest exercises are inserted until the student has become familiar with geometrical methods, and is furnished with elementary but much needed instruction in the art of handling original propositions; and he is assisted by diagrams and hints as long as these helps are necessary to develop his mental powers sufficiently to enable him to carry on the work by himself.

The law of converse theorems, the distinction between positive and negative quantities, and the principles of reciprocity and continuity have been briefly explained; but the application of these principles is left mainly to the discretion of teachers.

The author desires to express his appreciation of the valuable suggestions and assistance which he has received from distinguished educators in all parts of the country. He also desires to acknowledge his obligation to Mr. Charles Hamilton, the Superintendent of the composition room of the Athenæum Press, and to Mr. I. F. White, the compositor, for the excellent typography of the book.

Criticisms and corrections will be thankfully received.

G. A. WENTWORTH.

EXETER, N. H., June, 1899.

# NOTE TO TEACHERS.

It is intended to have the first fourteen pages of this book simply read in the class, with such running comment and discussion as may be useful to help the beginner catch the spirit of the subject-matter, and not leave him to the mere letter of dry definitions. In like manner, the definitions at the beginning of each Book should be read and discussed in the recitation room. There is a decided advantage in having the definitions for each Book in a single group so that they can be included in one survey and discussion.

For a similar reason the theorems of limits are considered together. The subject of limits is exceedingly interesting in itself, and it was thought best to include in the theory of limits in the second Book every principle required for Plane and Solid Geometry.

When the pupil is reading each Book for the first time, it will be well to let him write his proofs on the blackboard in his own language, care being taken that his language be the simplest possible, that the arrangement of work be vertical, and that the figures be accurately constructed.

This method will furnish a valuable exercise as a language lesson, will cultivate the habit of neat and orderly arrangement of work, and will allow a brief interval for deliberating on each step.

After a Book has been read in this way, the pupil should review the Book, and should be required to draw the figures free-hand. He should state and prove the propositions orally, using a pointer to indicate on the figure every line and angle named. He should be encouraged, in reviewing each Book, to do the original exercises; to state the converse propositions, and determine whether they are true or false; and also to give well-considered answers to questions which may be asked him on many propositions.

The Teacher is strongly advised to illustrate, geometrically and arithmetically, the principles of limits. Thus, a rectangle with a constant base $b$, and a variable altitude $x$, will afford an obvious illustration of the truth that the product of a constant and a variable is also a variable; and that the limit of the product of a constant and a variable is the product of the constant by the limit of the variable. If $x$ increases and approaches the altitude $a$ as a limit, the area of the rectangle increases and approaches the area of the rectangle $ab$ as a limit; if, however, $x$ decreases and approaches zero as a limit, the area of the rectangle decreases and approaches zero as a limit.

An arithmetical illustration of this truth may be given by multiplying the approximate values of any repetend by a constant. If, for example, we take the repetend 0.3333 etc., the approximate values of the repetend will be $\frac{3}{10}$, $\frac{33}{100}$, $\frac{333}{1000}$, $\frac{3333}{10000}$, etc., and these values multiplied by 60 give the series 18, 19.8, 19.98, 19.998, etc., which evidently approaches 20 as a limit; but the product of 60 into $\frac{1}{3}$ (the limit of the repetend 0.333 etc.) is also 20.

Again, if we multiply 60 into the different values of the decreasing series $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, etc., which approaches zero as a limit, we shall get the decreasing series 2, $\frac{1}{5}$, $\frac{1}{100}$, $\frac{1}{500}$, etc.; and this series evidently approaches zero as a limit.

The Teacher is likewise advised to give frequent written examinations. These should not be too difficult, and sufficient time should be allowed for accurately constructing the figures, for choosing the best language, and for determining the best arrangement.

The time necessary for the reading of examination books will be diminished by more than one half, if the use of symbols is allowed.

EXETER, N. H., 1899.

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## PLANE GEOMETRY.

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