

**REVOLVING VECTORS: WITH  
SPECIAL APPLICATION TO  
ALTERNATING CURRENT  
PHENOMENA**

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Revolving Vectors: With Special Application to Alternating Current Phenomena by George W. Patterson

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**GEORGE W. PATTERSON**

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## REVOLVING VECTORS



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# REVOLVING VECTORS

WITH SPECIAL APPLICATION TO

ALTERNATING CURRENT PHENOMENA

BY

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## PREFATORY NOTE

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THE use of complex quantities, i.e., quantities part real and part imaginary, in the theory of alternating currents has been greatly developed by Dr. Charles P. Steinmetz in his work on "Alternating Current Phenomena." It would be difficult to determine the influence which earlier writers, from the time of Caspar Wessel down to Steinmetz's day, have had in laying the foundation on which Steinmetz has built. It is, however, fair to say that the great advance in the use of vector methods, both algebraic and geometric, due to Dr. Steinmetz, justifies us in calling their application to alternating current phenomena Steinmetz's Method.

Earlier writers used complex quantities to represent *vector* quantities algebraically. Dr. Steinmetz extended the application so as to include *harmonic* quantities. As many writers on electrical subjects are prone to confuse vector and harmonic quantities, the author thinks it necessary to distinguish these two uses of complex quantities, and for that purpose he starts with the *vector* use and later takes up the *harmonic* use. In addition, subtraction and certain cases of multiplication and division, correct results are obtained by treating harmonic quantities as vector quantities; but in other cases of multiplication (such as multiplication of e.m.f. and current to obtain power) and division (such as dividing power by e.m.f. to get current) incorrect results are obtained unless arbi-

rary rules of multiplication and division are introduced. It therefore is necessary thoroughly to examine the fundamentals of these uses of complex quantities, and to deduce the laws of addition, subtraction, multiplication, and division, as applicable to vector quantities and to harmonic quantities whether simple (electromotive force and current) or compound (power), and also to such non-harmonic quantities as resistance, capacity, inductance, etc., in connection with harmonic quantities.

There are two methods employed by electricians using the complex quantity notation. The older method due to Dr. Steinmetz is expressed in graphical form by the wave diagram. The other method uses the so-called crank diagram. In both methods counter-clockwise rotations are used, though the formulæ have led some persons to think that Dr. Steinmetz has used clockwise rotations. It is true that the imaginary terms in the resulting formulæ have opposite signs. In reading Dr. Steinmetz's works no confusion need result for one accustomed to the crank diagram method if the differences are kept in mind. It has seemed to the author that the crank diagram method suits his purpose better, and consequently it will be used in this book.

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