ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS WITH EXAMPLES AND APPLICATIONS; PP. 1-271

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649571314

Elements of the Differential and Integral Calculus with Examples and Applications; pp. 1-271 by James M. Taylor

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

JAMES M. TAYLOR

ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS WITH EXAMPLES AND APPLICATIONS; PP. 1-271

Trieste

ELEMENTS

OF THE

DIFFERENTIAL AND INTEGRAL CALCULUS

WITH

EXAMPLES AND APPLICATIONS

BY

JAMES M. TAYLOR, A.M., LL.D
PROFESSOR OF MATHEMATICS
COLGATE UNIVERSITY

REVISED EDITION

BOSTON, U.S.A.
GINN & COMPANY, PUBLISHERS
The Athenæum Press
1902

PREFACE TO THE REVISED EDITION.

In this revision an attempt has been made to present in their unity the three methods commonly used in the Calculus. The concept of Rates is essential to a statement of the problems of the Calculus; the principles of Limits make possible general solutions of these problems, and the laws of Infinitesimals greatly abridge these solutions.

The Method of Rates, generalized and simplified, does not involve "the foreign element of time." For in measuring and comparing the rates of variables, the rate of any variable may be selected as the *unit of rates.* dy/dx is the x-rate of y, or the ratio of the rate of y to that of x, according as the rate of x is or is not the unit of rates.

The proofs of the principles of differentiation by the Method of Rates, and the numerous applications to geometry, mechanics, etc., found in Chapter II, render familiar the problem of rates before its solution by the Method of Limits or Infinitesimals is introduced.

In Chapter III, by proving that $\text{lt}\,(\Delta y/\Delta x) = dy/dx$, the problem of rates is reduced to the problem of finding the limit of the ratio of infinitesimals.

The Theory of Infinitesimals is that part of the Theory of Limits which treats of *variables having zero as their common limit.* In approaching its limit an infinitesimal passes through a series of finitely small values before it reaches infinitely small values. Infinitesimals can be divided into orders, and their laws can be established and applied when they are finitely small as well as when they are infinitely small. Hence, in the study of infinitesimals it is not necessary to determine that indefinite boundary between the finitely small and the infinitely small. Any small quantity *becomes* an infinitesimal when it *begins* to approach zero as its limit, not when it reaches any particular *degree* of *smallness*. A quantity, however small, which does not approach zero as its limit is not an infinitesimal. If it is recognized that the *essence* of infinitesimals lies in their *having zero as their limit*, rather than in their smallness, the study of them ceases to be mystical, obscure, and difficult.

Again, the concept of a limit as a constant whose value the variable never attains removes the necessity of studying the anatomy of Bishop Berkeley's "ghosts of departed quantities." Infinitesimals never equal zero and should not be denoted by the zero symbol. This distinction between infinitesimals and zero involves that between infinites and $a/0$.

The much-abused form $0/0$ cannot arise in the Calculus or elsewhere from any principle of limits; a distinctive service of the Theory of Limits is that it enables us to evaluate any determinate expression when it assumes this or any other indeterminate form.

Those who prefer to study the Calculus by the Method of Limits or Infinitesimals alone can omit the few demonstrations in Chapter II, which involve rates, and substitute for them the proofs by limits or infinitesimals in Chapter III.

To meet an increasing demand for a short course in differential equations, a chapter has been devoted to that subject.

A table of integrals arranged for convenience of reference is appended.

Throughout the work, as in previous editions, there are numerous practical problems from mechanics and other branches of applied mathematics which serve to exhibit the usefulness of the science, and to arouse and keep alive the interest of the student.

At the option of the teacher or reader, Chapters I and II of the Integral Calculus can be read after completing Chapters I and II of the Differential Calculus; also many of the numerous examples and problems may be omitted.

The author takes this opportunity of expressing his gratitude to the friends who by encouragement and suggestions have aided him in this revision.

JAMES M. TAYLOR.

HAMILTON, N. Y., 1898.

ELEMENTS OF THE CALCULUS.

PART I. DIFFERENTIAL CALCULUS.

CHAPTER I.

FUNCTIONS. RATES. DIFFERENTIALS.

1. A variable is a quantity which is, or is supposed to be, continually changing in value. Variable numbers are usually represented by the final letters of the alphabet, as x, y, z.

A constant is a quantity whose value is, or is supposed to be, fixed or invariable. Constant numbers may be *individual* or *general*. Individual constants are represented by figures; general constants are usually represented by the first letters of the alphabet.

For example, in the equation of the circle $(x-3)^2+(y-4)^2=r^2$, 3 and 4 are *individual* constants fixing the centre; r is a *general* constant denoting any radius; x and y are variables denoting the coördinates of the moving point which traces the circle.

2. Classification of variables. A variable whose value depends upon one or more other variables is called a *dependent variable*, or a *function* of those variables. A variable which does not depend upon any other variable is called an *independent variable*.

For example, x^3-b^2, $\sin x$, and $\log(x-a)$ are functions of the independent variable x. Again, $x^3+3xy+y^2$, $\log(x^2-y^2)$, and a^{x+y} are functions of the two variables x and y.

3. Classification of functions. An *algebraic* function is one which without the use of infinite series can be expressed by the operations of addition, subtraction, multiplication, division, and the operations denoted by constant exponents. All functions which are not algebraic are called *transcendental.* Of these, the more common are:

The *exponential* function $y = a^x$; and its inverse, the *logarithmic* function $x = \log_a y$.

The *trigonometric* functions $y = \sin x$, $y = \cos x$, etc.; and the *inverse-trigonometric* functions $x = \sin^{-1} y$, $x = \cos^{-1} y$, etc.

4. Explicit and implicit functions. When an equation involving two or more variables is solved for any one of them, this one is said to be an *explicit* function of the others. When an equation is not so solved, any one of its variables is called an *implicit* function of the others.

For example, in $x^2 + y^2 = r^2$, either y or x is an *implicit* function of the other; while in $y = \pm \sqrt{r^2 - x^2}$, y is an explicit function of x, and in $x = \pm \sqrt{r^2 - y^2}$, x is an explicit function of y.

A function is said to be one-valued, two-valued, or n-valued according as for each value of its variable it has one value, two values, or n values.

For example, y is a one-valued function in $y = x^3$, a two-valued function in $y^2 = 4px$, and a three-valued function in $y^3 + xy + x^2 = 1$.

5. Increasing and decreasing functions. An *increasing* function is one which *increases* when its variable increases. Hence, it decreases when its variable decreases.

A *decreasing* function is one which *decreases* when its variable *increases.* Hence, it increases when its variable decreases.

For example, $5x$ and $\log x$ are increasing functions of x, while $-5x$ and $1/x$ are decreasing functions of x.