INTRODUCTION TO THE THEORY OF ALGEBRAIC EQUATIONS

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BY

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PREFACE.

THE solution of the general quadratic equation was known as early as the ninth century; that of the general cubic and quartic equations was discovered in the sixteenth century. During the succeeding two centuries many unsuccessful attempts were made to solve the general equations of the fifth and higher degrees. In 1770 Lagrange analyzed the methods of his predecessors and traced all their results to one principle, that of rational resolvents, and proved that the general quintic equation cannot be solved by rational resolvents. The impossibility of the algebraic solution of the general equation of degree n $(n>4)$, whether by rational or irrational resolvents, was then proved by Abel, Wantzel, and Galois. Out of these algebraic investigations grew the theory of substitutions and groups. The first systematic study of substitutions was made by Cauchy (*Journal de l'école polytechnique*, 1815).

The subject is here presented in the historical order of its development. The First Part (pp. 1–41) is devoted to the Lagrange-Cauchy-Abel theory of general algebraic equations. The Second Part (pp. 42–98) is devoted to Galois' theory of algebraic equations, whether with arbitrary or special coefficients. The aim has been to make the presentation strictly elementary, with practically no dependence upon any branch of mathematics beyond elementary algebra. There occur numerous illustrative examples, as well as sets of elementary exercises.

In the preparation of this book, the author has consulted, in addition to various articles in the journals, the following treatises;

Lagrange, *Réflexions sur la résolution algébrique des équations*; Jordan, *Traité des substitutions et des équations algébriques*; Serret, *Cours d'Algèbre supérieure*; Netto-Cole, *Theory of Substitutions and its Applications to Algebra*; Weber, *Lehrbuch der Algebra*; Burnside, *The Theory of Groups* Pierpont, *Galois' Theory of Algebraic Equations*, Annals of Math., 2d ser., vols. 1 and 2; Bolza, *On the Theory of Substitution-Groups and its Applications to Algebraic Equations*, Amer. Journ. Math., vol. XIII.

The author takes this opportunity to express his indebtedness to the following lecturers whose courses in group theory he has attended: Oscar Bolza in 1894, E. H. Moore in 1895, Sophus Lie in 1896, Camille Jordan in 1897.

But, of all the sources, the lectures and publications of Professor Bolza have been of the greatest aid to the author. In particular, the examples (§ 65) of the group of an equation have been borrowed with his permission from his lectures.

The present elementary presentation of the theory is the outcome of lectures delivered by the author in 1897 at the University of California, in 1899 at the University of Texas, and twice in 1902 at the University of Chicago.

CHICAGO, *August*, 1902

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1. Quadratic equation. The roots of $x^2+px+q=0$ are

$$x_1=\tfrac{1}{2}(-p+\sqrt{p^2-4q}), \quad x_2=\tfrac{1}{2}(-p-\sqrt{p^2-4q}).$$

By addition, subtraction, and multiplication, we get

$$x_1+x_2=-p, \quad x_1-x_2=\sqrt{p^2-4q}, \quad x_1x_2=q.$$

Hence the irrationality $\sqrt{p^2-4q}$, which occurs in the expressions for the roots, is rationally expressible in terms of the roots, being equal to x_1-x_2. Unlike the last function, the functions x_1+x_2 and x_1x_2 are *symmetric* in the roots and are rational functions of the coefficients.

2. Cubic equation. The general cubic equation may be written

(1) $$x^3-c_1x^2+c_2x-c_3=0.$$

Setting $x=y+\frac{1}{3}c_1$, the equation (1) takes the simpler form

(2) $$y^3+py+q=0,$$

* *Réflexions sur la résolution algébrique des équations*, Œuvres de Lagrange, Paris, 1869, vol. 3; first printed by the Berlin Academy, 1770–71.