

# **PRACTICAL LEAST SQUARES**

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Practical least squares by Ora Miner Leland

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**ORA MINER LELAND**

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LEAST SQUARES**



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BY

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ASTRONOMY DEPT.

TO  
MY WIFE  
WHOSE LOYAL ASSISTANCE  
HAS CONTRIBUTED IN GREAT MEASURE  
TO ITS PREPARATION,  
THIS BOOK IS AFFECTIONATELY  
DEDICATED

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## PREFACE

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THIS book results from the author's experience in teaching the subject of Least Squares and the Adjustment of Observations to classes of civil engineering students at Cornell University. As the time allotted to this work became more and more limited, the available textbooks became less adaptable to the scope of the course. To meet this condition, a series of chapters entitled "Notes on the Adjustment of Observations" was prepared and used as a text. With these notes as a basis, this book has been written.

It is designed particularly for use in short courses of instruction and by engineers and scientists in connection with their private practice. It will not replace the more elaborate treatises on the subject but the author hopes that it will introduce the student directly to the simpler methods of solving the ordinary problems in adjustment.

The plan of the work is essentially practical. After a general introduction devoted to a consideration of the character and occurrence of errors, the adjustment of direct, indirect, and conditioned observations is taken up in detail and illustrated by numerical applications to triangulation, leveling, astronomy, and the derivation of empirical formulas. Not until after this practical treatment of the determination of the best values of the unknown quantities is the precision of observations discussed, together with the computation of the mean square and probable errors of the observations and results. Finally, the principles of probability and the analytical derivation of the Law of Error are given in appendices.

The utility of this arrangement should be obvious. By far the greater number of applications of Least Squares do not require a consideration of the precision of the results nor a knowledge of the mean square or probable errors. Moreover,

the subject of the precision is usually the most troublesome part of the work for the student or the beginner to understand. Therefore, the practical methods of adjustment are explained directly and fully, without regard to the probable errors or to the theoretical derivation of the Law of Error. A special effort has been made to explain the procedure in each case as completely as necessary for the beginner as well as the practitioner, even at the risk of criticism for undue length. The usual difficulties experienced by students seem to justify this effort.

In Appendix D there is given an outline of a short course of instruction suitable for civil engineers. This plan was carried out successfully by the author in sixteen lessons. While it is not at all desirable to restrict the work so severely, if no more time can be given to it the course is still very much worth while.

The author is indebted to many excellent works and has endeavored to make specific acknowledgments throughout the book wherever due. In the preparation of the original notes and their application to class instruction, his thanks are due to his former colleagues, Professors P. H. Underwood and L. A. Lawrence, for their assistance and suggestions.

O. M. LELAND.

MINNEAPOLIS, MINN.

*September, 1921.*

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