

**LECTURES ON FUNDAMENTAL
CONCEPTS OF ALGEBRA AND
GEOMETRY; WITH A NOTE ON
THE GROWTH OF ALGEBRAIC
SYMBOLISM**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649628308

Lectures on Fundamental Concepts of Algebra and Geometry; With a Note on the Growth of Algebraic Symbolism by John W. Young & William W. Denton & U. G. Mitchell

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

JOHN W. YOUNG & WILLIAM W. DENTON & U. G. MITCHELL

**LECTURES ON FUNDAMENTAL
CONCEPTS OF ALGEBRA AND
GEOMETRY; WITH A NOTE ON
THE GROWTH OF ALGEBRAIC
SYMBOLISM**

**LECTURES ON FUNDAMENTAL CONCEPTS
OF ALGEBRA AND GEOMETRY**



THE MACMILLAN COMPANY

NEW YORK · BOSTON · CHICAGO
SAN FRANCISCO

MACMILLAN & CO., LIMITED

LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

LECTURES
ON
FUNDAMENTAL CONCEPTS OF
ALGEBRA AND GEOMETRY

BY

JOHN WESLEY YOUNG

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF KANSAS

PREPARED FOR PUBLICATION WITH THE
COÖPERATION OF

WILLIAM WELLS DENTON

ASSISTANT IN MATHEMATICS IN THE UNIVERSITY OF ILLINOIS

WITH A NOTE ON

THE GROWTH OF ALGEBRAIC SYMBOLISM

BY

ULYSSES GRANT MITCHELL

ASSISTANT PROFESSOR OF MATHEMATICS
IN THE UNIVERSITY OF KANSAS

STANFORD LIBRARY

New York

THE MACMILLAN COMPANY

1911

All rights reserved

K

COPYRIGHT, 1911,
By THE MACMILLAN COMPANY.

Set up and electrotyped. Published June, 1911.

161662

YVARELLI GROFNATZ

Notional Press
J. B. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

THE following lectures contain an elementary account of the logical foundations of algebra and geometry,—elementary, in the sense that the technical mathematical equipment presupposed on the part of the reader has been reduced to a minimum. Except in a very few instances, no knowledge of mathematics beyond the most elementary portions of algebra and geometry has been assumed. It has been my purpose to give a general exposition of the abstract, formal point of view developed during the last few decades, rather than an exhaustive treatment of the details of the investigations.

The results of recent work on the logical foundations are of vital interest alike to the teachers of mathematics in our secondary schools and colleges and to philosophers and logicians. I hope that both these classes will welcome a concise statement of some of the more fundamental of these results and an elementary exposition, omitting all involved details, of the point of view which governs all present work on the foundations. The book should be available also as a text in connection with so-called Teachers' Courses in colleges and universities.

The lectures were given at the University of Illinois during the summer of 1909. They are here reproduced in

substantially the same form as delivered. The conversational style has, to a large extent, been retained in the hope that the presentation has gained thereby in spontaneity.

My cordial thanks are due my former colleagues, Dean E. J. Townsend and Professor G. A. Miller, of the University of Illinois, who read the greater part of the manuscript; and to my colleague, Professor U. G. Mitchell, who not only read the whole manuscript and rendered valuable assistance in seeing the book through the press, but has added to its value by contributing the Note on "The Growth of Algebraic Symbolism," which will be found at the end of the lectures. Above all, however, my thanks are due to Mr. W. W. Denton, of the University of Illinois, without whose help the lectures would probably not have been published. He took the lectures down stenographically, and applied himself to the revision of the resulting manuscript with great enthusiasm and keen insight.

J. W. YOUNG.

LAWRENCE, KANSAS,
April, 1911.

CONTENTS

CHAPTER	PAGE
I. INTRODUCTION. EUCLID'S ELEMENTS	1
II. A NON-EUCLIDEAN WORLD	14
III. ON THE HISTORY OF THE PARALLEL POSTULATE	26
IV. LOGICAL SIGNIFICANCE OF DEFINITIONS, AXIOMS, AND POSTULATES	36
V. CONSISTENCY, INDEPENDENCE, AND CATEGORICALNESS OF A SET OF ASSUMPTIONS	43
VI. CLASS CORRESPONDENCE NUMBER	58
VII. ORDER. DISCRETE SEQUENCES	67
VIII. THE SEQUENCE OF CARDINAL NUMBERS. DENUMERABLE CLASSES. DENSE CLASSES. CONTINUOUS CLASSES	79
IX. GROUP. NUMBER SYSTEM	88
X. HISTORICAL AND LOGICAL DEVELOPMENT OF THE CON- CEPT OF REAL NUMBER	99
XI. NEGATIVE NUMBERS	107
XII. ORDINARY AND HIGHER COMPLEX NUMBERS	121
XIII. GEOMETRY. HILBERT'S ASSUMPTIONS	134
XIV. HILBERT'S ASSUMPTIONS (<i>Continued</i>)	146
XV. PIERI'S ASSUMPTIONS	155
XVI. THE DIMENSIONS OF A CLASS	166
XVII. SPACES OF FOUR OR MORE DIMENSIONS	178
XVIII. ALGEBRA AND GEOMETRY	181
XIX. VARIABLE AND FUNCTION	192
XX. LIMIT	201
XXI. GENERAL CONCLUSIONS	215
NOTE. THE GROWTH OF ALGEBRAIC SYMBOLISM, BY U. G. MITCHELL	226