PLANE TRIGONOMETRY

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Plane trigonometry by G. A. Wentworth

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PLANE TRIGONOMETRY

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PLANE TRIGONOMETRY

BY

G. A. WENTWORTH, A.M.,

PROFESSOR OF MATHEMATICS IN PHILLIPS EXETER ACADEMY

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PREFACE.

IN preparing this work the aim has been to furnish just so much of Trigonometry as is actually taught in our best schools and colleges. Consequently, development of functions in series and all other investigations that are important only for the special student have been omitted. The principles have been unfolded with the utmost brevity consistent with simplicity and clearness, and interesting problems have been selected with a view to awaken a real love for the study. Much time and labor have been spent in devising the simplest proofs for the propositions, and in exhibiting the best methods of arranging the logarithmic work.

The object of the work on Surveying and Navigation is to present these subjects in a clear and intelligible way, according to the best methods in actual use; and also to present them in so small a compass that students in general may find the time to acquire a competent knowledge of these very interesting and important studies.

The author is under particular obligation for assistance to G. A. Hill, A.M., of Cambridge, Mass., and to Prof. James L. Patterson, of Lawrenceville, N.J.

G. A. WENTWORTH.

PHILLIPS EXETER ACADEMY,
September, 1882.

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CONTENTS.

PLANE TRIGONOMETRY.

CHAPTER I. Functions of Acute Angles:

Definitions, 1; representation of functions by lines, 7; changes in the functions as the angle changes, 9; functions of complementary angles, 10; relations of the functions of an angle, 11; formulas for finding all the other functions of an angle when one function of the angle is given, 13; functions of 45°, 30°, 60°, 15.

CHAPTER II. The Right Triangle:

Solution: Case I., when an acute angle and the hypotenuse are given, 16; Case II., when an acute angle and the opposite leg are given, 17; Case III., when an acute angle and the adjacent leg are given, 17; Case IV., when the hypotenuse and a leg are given, 18; Case V., when the two legs are given, 18; general method of solving the right triangle, 19; area of the right triangle, 20; the isosceles triangle, 24; the regular polygon, 26.

CHAPTER III. Goniometry:

Definition of Goniometry, 28; angles of any magnitude, 28; general definitions of the functions of angles, 29; algebraic signs of the functions, 31; functions of a variable angle, 32; functions of angles larger than 360°, 34; formulas for acute angles extended to all angles, 35; reduction of the functions of all angles to the functions of angles in the first quadrant, 38; functions of angles that differ by 90°, 40; functions of a negative angle, 41; functions of the sum of two angles, 43; functions of the difference of two angles, 45; functions of twice an angle, 47; functions of half an angle, 47; sums and differences of functions, 48.

CHAPTER IV. The Oblique Triangle:

Law of sines, 50; law of cosines, 52; law of tangents, 52. Solution: Case I., when one side and two angles are given, 54; Case II., when two sides and the angle opposite to one of them are given, 56; Case III., when two sides and the included angle are given, 60; Case IV., when the three sides are given, 64; area of a triangle, 68; miscellaneous problems, 71–88.

EXAMINATION PAPERS, 89–102.

SPHERICAL TRIGONOMETRY.

CHAPTER V. The Right Spherical Triangle:

Introduction, 103; formulas relating to right spherical triangles, 105; Napier's rules, 108. Solution: Case I., when the two legs are given, 110; Case II., when the hypotenuse and a leg are given, 110; Case III., when a leg and the opposite angle are given, 111; Case IV., when a leg and an adjacent angle are given, 111; Case V., when the hypotenuse and an oblique angle are given, 111; Case VI., when the two oblique angles are given, 111; solution of the isosceles spherical triangle, 116; solution of a regular spherical polygon, 116.

CHAPTER VI. The Oblique Spherical Triangle:

Fundamental formulas, 117; formulas for half angles and sides, 119; Gauss's equations and Napier's analogies, 121. Solution: Case I., when two sides and the included angle are given, 123; Case II., when two angles and the included side are given, 125; Case III., when two sides and an angle opposite to one of them are given, 127; Case IV., when two angles and a side opposite to one of them are given, 129; Case V., when the three sides are given, 130; Case VI., when the three angles are given, 131; area of a spherical triangle, 133.

CHAPTER VII. Applications of Spherical Trigonometry:

Problem, to reduce an angle measured in space to the horizon, 136; problem, to find the distance between two places on the earth's surface when the latitudes of the places and the difference of their longitudes are known, 137; the celestial sphere, 137; spherical co-ordinates, 140; the astronomical triangle, 142; astronomical problems, 143–146.

PLANE TRIGONOMETRY.

CHAPTER I.

TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES.

§ 1. Definitions.

The sides and angles of a plane triangle are so related that any three given parts, provided at least one of them is a side, determine the shape and the size of the triangle.

Geometry shows how, from three such parts, to *construct* the triangle and find the values of the unknown parts.

Trigonometry shows how to *compute* the unknown parts of a triangle from the numerical values of the given parts.

Geometry shows in a general way that the sides and angles of a triangle are mutually dependent. Trigonometry begins by showing the exact nature of this dependence in the *right triangle*, and for this purpose employs the *ratios of its sides*.

Let MAN (Fig. 1) be an acute angle. If from any points B, D, F, ... in one of its sides perpendiculars BC, DE, FG, ... are let fall to the other side, then the right triangles ABC, ADE, AFG, ... thus formed have the angle A common, and are therefore mutually equiangular and similar. Hence, the ratios of their corresponding sides, pair by pair, are equal. That is,

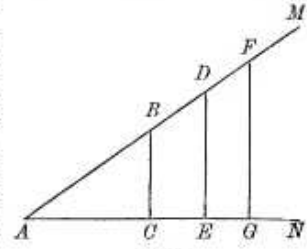

Fig. 1.

$$\frac{AC}{AB}=\frac{AE}{AD}=\frac{AG}{AF}; \quad \frac{AC}{BC}=\frac{AE}{DE}=\frac{AG}{FG}; \text{ etc.}$$