OCTONIONS: A DEVELOPMENT OF CLIFFORD'S BI-QUATERIONS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649004300

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Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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Trieste

A TREATISE

ON

OCTONIONS.

London: C. J. CLAY AND SONS,
CAMBRIDGE UNIVERSITY PRESS WAREHOUSE,
AVE MARIA LANE.
Glasgow: 263, Argyle Street.

Leipzig: F. A. BROCKHAUS.
New York: THE MACMILLAN COMPANY.
Bombay: E. SEYMOUR HALE.

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BY

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CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1898

Cambridge:
PRINTED BY J. AND C. F. CLAY,
AT THE UNIVERSITY PRESS.

PREFACE.

I OWE a great debt of gratitude to an old pupil for the results of a casual conversation I had with him some six or seven years ago. On that occasion Mr P. A. M. Parker discoursed of rotors and motors in such wise that it seemed to his tutor high time to rub the dust from the volume of Clifford's *Mathematical Papers* lying on the shelves; for otherwise the tutor and pupil bade fair to change places. Many days of most interesting work and thought have been the sequel of that talk within the walls of Ormond College.

The treatment below of what Clifford called Bi-quaternions runs on two sharply-defined lines. Quaternions and the Ausdehnungslehre have both been pressed into the service, and the help from them has led to very different kinds of development. Neither development could, in my opinion, be well spared. The first seems to be allied to metrical geometry and the second to descriptive. At any rate I do not see how, in few words, better to describe the essential characteristics of the two. For more precise ideas the reader must study the subject itself.

So far as the present treatise is concerned, these developments took place in two periods. I had done what I could on the quaternion model, but being dissatisfied because so many questions which presented themselves were thereby but imperfectly answered, I put the work aside. Meanwhile I had been led by Sir Robert Ball's *Theory of Screws* (Dublin, 1876) to study the *Ausdehnungslehre* (1862), and was delighted to find that the gaps could apparently be filled from this source. On taking the subject up again it was found that this surmise was correct.

Perhaps the most striking fact that has come to light in the investigation is one that appeared almost at the outset, and one which mainly induced me to proceed. I mean the fact that every quaternion formula except such as involve ∇ admits of an octonion interpretation—a geometrical interpretation much more general than that which it was primarily meant to have. There is matter for reflection in that the founders of Quaternions, while they were busying themselves only with vector and quaternion conceptions, were, all the time, unknown to themselves, establishing motor and octonion truths.

There is a corresponding though less striking fact connected with the second method of development. The statements of Quaternions were always intended to have but one meaning—of course with many variations of form when put into words—but the Ausdehnungslehre was intended to be a general framework of symbols whose applications should be in many provinces of thought. It is therefore not surprising that there are geometrical interpretations which were not developed, even if seriously contemplated, by Grassmann. Such are the applications of the Ausdehnungslehre below. I cannot believe that Grassmann contemplated such applications of his calculus if only because apparently he never conceived of a magnitude other than zero whose "numerical value," in his own technical sense, was zero.

It may be asked why, in this treatise, I start *de novo*, instead of taking all that Clifford has done for granted. The reasons are (1) the desirability of making the treatise self-contained; (2) the fact that Clifford uses a method dependent on the properties of non-Euclidean space, whereas I regard the subject as referring to Euclidean space; and (3) I do not altogether understand all of Clifford's arguments.

The treatise suffers in form, somewhat, from the fact that it was not, in the making, meant to be a book but a "paper," as will be directly explained. If I had from the beginning contemplated the book form, or if when the treatise became destined to take that form, I had thought myself justified, or indeed had had the courage, to recast the whole appropriately, it would have been at least doubled in length, without probably any material mathematical amplification. Not only is it too condensed, where the argument is fairly covered, but many steps of reasoning are left out which the reader will require patience to supply, as I myself have found in reading the proof-sheets. I can but apologise to the reader for these rather irritating defects.

There is a defect wholly unconnected with this, which qualified critics may help to remedy. I refer to the terminology. I have found myself compelled to invent quite a little vocabulary; and if ever there was an author in such an uncomfortable position whose ignorance of dead and other languages was more profound than the writer's, I pity him. About the term "Octonion" I shall speak directly. The three groups of terms *augmenter, tensor, additor, pitch*; and *twister, versor, translator*; and *velocity motor, force motor, momentum motor*; and their congeners I am (pending criticism) content with. The group of terms referring to linear motor functions of motors; *general function, commutative function, pencil function, energy function*; are passable. The terms *convert, convertor, axial quaternion* (or *axial*) and some less frequently used seem to me like unwilling conscripts begging at any price for substitutes. The term *variation* as used in the treatise is objectionable on the ground that it clashes with the technical meaning of the same term in Algebra. If I had thought that any serious inconvenience would result I should have used some such term as *replacement*, but I thought *variation* better. [It must be