

**ELEMENTS OF DYNAMIC; AN  
INTRODUCTION TO THE STUDY  
OF MOTION AND REST IN SOLID  
AND FLUID BODIES. PART I.  
KINEMATIC**

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Part I. Kinematic by W. K. Clifford

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**W. K. CLIFFORD**

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ELEMENTS OF DYNAMIC.



# ELEMENTS OF DYNAMIC

AN INTRODUCTION TO THE STUDY OF

MOTION AND REST

IN SOLID AND FLUID BODIES

BY

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