

**ELEMENTS OF THE  
DIFFERENTIAL AND  
INTEGRAL CALCULUS**

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Elements of the Differential and Integral Calculus by Charles Davies

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**CHARLES DAVIES**

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ELEMENTS  
OF THE  
DIFFERENTIAL AND INTEGRAL  
CALCULUS.

BY CHARLES DAVIES,  
AUTHOR OF MENTAL AND PRACTICAL ARITHMETIC, FIRST LESSONS IN  
ALGEBRA, ELEMENTS OF SURVEYING, ELEMENTS OF DESCRIPTIVE  
GEOMETRY, ELEMENTS OF ANALYTICAL GEOMETRY,  
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HENRY W. BEEB, STENOGRAPHER,  
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## PREFACE.

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THE Differential and Integral Calculus is justly considered the most difficult branch of the pure Mathematics.

The methods of investigation are, in general, not as obvious, nor the connection between the reasoning and the results so clear and striking, as in Geometry, or in the elementary branches of analysis.

It has been the intention, however, to render the subject as plain as the nature of it would admit, but still, it cannot be mastered without patient and severe study.

This work is what its title imports, an Elementary Treatise on the Differential and Integral Calculus. It might have been much enlarged, but being intended for a text-book, it was not thought best to extend it beyond its present limits.

The works of Boucharlat and Lacroix have been freely used, although the general method of arranging the subjects is quite different from that adopted, by either of those distinguished authors

MILITARY ACADEMY,  
*West Point, October, 1836.*



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