INTRODUCTION TO INFINITESIMAL ANALYSIS: FUNCTIONS OF ONE REAL VARIABLE

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INFINITESIMAL ANALYSIS

FUNCTIONS OF ONE REAL VARIABLE

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PREFACE.

A COURSE dealing with the fundamental theorems of infinitesimal calculus in a rigorous manner is now recognized as an essential part of the training of a mathematician. It appears in the curriculum of nearly every university, and is taken by students as "Advanced Calculus" in their last collegiate year, or as part of "Theory of Functions" in the first year of graduate work. This little volume is designed as a convenient reference book for such courses; the examples which may be considered necessary being supplied from other sources. The book may also be used as a basis for a rather short theoretical course on real functions, such as is now given from time to time in some of our universities.

The general aim has been to obtain rigor of logic with a minimum of elaborate machinery. It is hoped that the systematic use of the Heine-Borel theorem has helped materially toward this end, since by means of this theorem it is possible to avoid almost entirely the sequential division or "pinching" process so common in discussions of this kind. The definition of a limit by means of the notion "value approached" has simplified the proofs of theorems, such as those giving necessary and sufficient conditions for the existence of limits, and in general has largely decreased the number of ϵ's and δ's. The theory of limits is developed for multiple-valued functions, which gives certain advantages in the treatment of the definite integral.

In each chapter the more abstract subjects and those which can be omitted on a first reading are placed in the concluding sections. The last chapter of the book is more advanced in character than the other chapters and is intended as an introduction to the study of a special subject. The index at the end of the book contains references to the pages where technical terms are first defined.

When this work was undertaken there was no convenient source in English containing a rigorous and systematic treatment of the body of theorems usually included in even an elementary course on real functions, and it was necessary to refer to the French and German treatises. Since then one treatise, at least, has appeared in English on the Theory of Functions of Real Variables. Nevertheless it is hoped that the present volume, on account of its conciseness, will supply a real want.

The authors are much indebted to Professor E. H. Moore of the University of Chicago for many helpful criticisms and suggestions; to Mr. E. B. Morrow of Princeton University for reading the manuscript and helping prepare the cuts; and to Professor G. A. Bliss of Princeton, who has suggested several desirable changes while reading the proof-sheets.

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