

**PLANE AND SPHERICAL  
TRIGONOMETRY  
AND MENSURATION**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649672271

Plane and Spherical Trigonometry and Mensuration by A. Schuyler

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**A. SCHUYLER**

**PLANE AND SPHERICAL  
TRIGONOMETRY  
AND MENSURATION**



PLANE AND SPHERICAL  
TRIGONOMETRY

AND

MENSURATION.

BY

A. SCHUYLER, LL. D.,

*Professor of Mathematics and Philosophy in Kansas Wesleyan University;  
Author of Principles of Logic, Empirical and Rational Psychology  
and of a Series of Mathematical Works.*

---

NEW-YORK ❖ CINCINNATI ❖ CHICAGO  
AMERICAN BOOK COMPANY

*Educ. P. 90 118*

WARVARD COLLEGE LIBRARY

By exchange from

BERLIN COLLEGE LIBRARY

*Jan 30 1875*

---

COPYRIGHT

1875

BY WILSON, HINKLE & Co.

---

L-P 18

## PREFACE.

---

THE following treatise on Plane and Spherical Trigonometry and Mensuration first appeared in 1873 as an Introduction to the author's work on Surveying and Navigation.

The general favor with which the Trigonometry has been received renders it probable that its publication in a separate volume would greatly extend its usefulness.

The many studying Trigonometry, who do not care to learn Surveying, ought not to be compelled to purchase a more expensive book than is necessary.

In the application of Logarithms to the processes of Multiplication and Division, Involution and Evolution, the order of treatment is, first, the proposition and its demonstration; next, the rule, then the solution of examples, thus giving the application of the principle in immediate connection with its statement.

The trigonometrical functions are defined, not as ratios, but as linear functions of the angle, thus giving the student clear geometrical conceptions instead of abstract relations, and enabling him the more readily to grasp the laws of the algebraic signs of the functions. The advantages in analytic investigations resulting from defining these functions as ratios have been secured in the principles relating to the Right Triangle, Art. 64.

Each of the circular functions has, in the first place, been considered by itself, and its value traced for all arcs, from  $0^\circ$  to  $360^\circ$ .

Trigonometry is naturally divided into Plane and Spherical. In Plane Trigonometry triangles are discussed in the order, *Right Triangles* and *Oblique Triangles*.

Then, under the general head, *Relations of the Circular Func-*

tions, follow, fundamental formulas, each function in terms of each of the others, functions of negative arcs, functions of  $(n 90^\circ \mp a)$ , values of functions of particular arcs, inverse functions, functions of the sum and difference of two angles, functions of double and half angles, consequences of the formulas (a), (b), (c), (d), a variety of interesting practical applications, and the computation of the natural and logarithmic functions.

In Spherical Trigonometry, as in Plane, *Right Triangles* are first discussed, then *Oblique*.

More than ordinary care has been given to the development of Napier's principles and to the discussion of the species of the parts of both right and oblique triangles, Arts. 126, 129, 145, 148, 151.

Special attention is invited to Arts. 64, 89, 91, 126, 129, 145, 148.

Mensuration, a subject at once interesting and practically important, has been discussed at length, and formulas, instead of rules, have been developed for the solution of problems.

Hoping that the work, as a whole, will prove a contribution to the wants of the student, and render him efficient aid in acquiring a correct mathematical taste, and that its publication in a separate volume will greatly extend its usefulness, it is submitted to the favorable consideration of those who have the responsibility of selecting the text-book on this important branch of mathematical science.

A. SCHUYLER.

BALDWIN UNIVERSITY,  
Berea, O., June 12, 1875. }



## CONTENTS.

---

	PAGE
INTRODUCTION. . . . .	9
Definition of a logarithm. . . . .	9
Common logarithms. . . . .	9
Laws of the characteristic. . . . .	10
Exercises on the characteristic. . . . .	11
Description of the table of logarithms. . . . .	12
To find the logarithm of a number. . . . .	13
To find the number corresponding to a logarithm. . . . .	16
Multiplication by logarithms. . . . .	18
Division by logarithms. . . . .	19
Arithmetical complement. . . . .	20
Involution by logarithms. . . . .	21
Evolution by logarithms. . . . .	22
TRIGONOMETRY. . . . .	23
PLANE TRIGONOMETRY. . . . .	23
Trigonometrical functions. . . . .	27
The sine of an arc. . . . .	29
The co-sine of an arc. . . . .	30
The versed-sine of an arc. . . . .	32
The co-versed-sine of an arc. . . . .	33
The tangent of an arc. . . . .	34
The co-tangent of an arc. . . . .	35
The secant of an arc. . . . .	37
The co-secant of an arc. . . . .	38
Signs of the circular functions. . . . .	40
Limiting values of the circular functions. . . . .	40
Natural functions. . . . .	41

	PAGE
Logarithmic functions. . . . .	43
<i>Right Triangles</i> —Principles. . . . .	47
Case I. . . . .	51
Case II. . . . .	52
Case III. . . . .	53
Case IV. . . . .	54
<i>Oblique Triangles</i> .—Case I. . . . .	55
Case II. . . . .	58
Case III. . . . .	64
Case IV. . . . .	67
<i>Heights and Distances</i> . . . . .	69
Problems. . . . .	70
Fundamental formulas. . . . .	72
Each function in terms of the others. . . . .	76
Functions of negative arcs. . . . .	78
Functions of $(n 90^\circ \mp a)$ . . . . .	78
Values of functions of particular arcs. . . . .	80
Inverse trigonometric functions. . . . .	81
Sine and co-sine of the sum of two angles. . . . .	83
Sine and co-sine of the difference of two angles. . . . .	85
Tangent and co-tangent of the sum or difference. . . . .	86
Functions of double and half angles. . . . .	88
Consequences of $(a), (b), (c), (d)$ . . . . .	90
Applications. . . . .	92
Miscellaneous exercises. . . . .	106
Computation of functions. . . . .	107
SPHERICAL TRIGONOMETRY. . . . .	108
<i>Right Triangles</i> .—Napier's circular parts. . . . .	109
Napier's principles. . . . .	110
Maudnit's principles. . . . .	112
Analogies of plane and spherical triangles. . . . .	113
Species of the parts. . . . .	114
Remarks. . . . .	119
Polar triangles. . . . .	122
Quadrantal triangles. . . . .	123
<i>Oblique Triangles</i> .—Proposition I. . . . .	124
Proposition II. . . . .	125

CONTENTS.

	vii PAGE
Propositions III and IV. . . . .	126
Propositions V and VI. . . . .	128
Proposition VII. . . . .	129
Propositions VIII and IX. . . . .	130
Napier's analogies. . . . .	131
Values of $h$ as $b$ increases from $0^\circ$ to $360^\circ$ . . . . .	133
Case I. . . . .	135
Values of $P$ as $B$ increases from $0^\circ$ to $360^\circ$ . . . . .	138
Case II. . . . .	140
Principles. . . . .	144
Case III. . . . .	147
Cases IV and V. . . . .	148
Case VI. . . . .	149
 MENSURATION. . . . .	 150
MENSURATION OF SURFACES. . . . .	150
Area of a rectangle. . . . .	150
Area of a parallelogram. . . . .	151
Area of a triangle. . . . .	151
Area of a quadrilateral. . . . .	154
Area of an irregular polygon. . . . .	159
Area of a regular polygon. . . . .	161
Formulas for the circle. . . . .	163
Area of a sector of a circle. . . . .	164
Area of a segment of a circle. . . . .	165
Area of an ellipse. . . . .	166
Area of a regular prism. . . . .	166
Area of a regular pyramid. . . . .	167
Area of a frustum of a regular pyramid. . . . .	168
Area of a cylinder. . . . .	168
Area of a cone. . . . .	169
Area of a frustum of a cone. . . . .	169
Area of a sphere. . . . .	170
Area of a zone. . . . .	171
Area of a spherical triangle. . . . .	171
Area of a spherical polygon. . . . .	172
Area of a regular polyhedron. . . . .	173