ELEMENTARY COURSE IN LAGRANGE'S EQUATIONS AND THEIR APPLICATIONS TO SOLUTIONS OF PROBLEMS OF DYNAMICS

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Elementary Course in Lagrange's Equations and Their Applications to Solutions of Problems of Dynamics by N. W. Akímoff

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ELEMENTARY COURSE

IN

LAGRANGE'S EQUATIONS

AND

THEIR APPLICATIONS TO SOLUTIONS OF PROBLEMS OF DYNAMICS

WITH NUMEROUS EXAMPLES

BY

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PREFACE.

In compiling this work I have freely drawn on all best known books of such authorities as Appell, Routh, Loney, Bouasse and many others. My object is to present the matter in an elementary form and to make it immediately intelligible to a reader possessing but a limited knowledge of mathematics. For this reason it was thought necessary to explain things, already almost evident; constantly to refer back to elementary books on mechanics, etc.; and to devote the whole first chapter to elements, which are no doubt already familiar to the average reader, but which he may find presented in a somewhat different form from that in which he has them fixed in his mind.

Lagrange's method is like a slide rule: it has its limitations, yet, in many problems it enables us to write down the differential equations of motion almost instantly.

The wonderful beauty and power of this method will undoubtedly appeal to the reader, engineer or student, and make him *like* the whole subject of dynamics, although his teachers may have completely failed even to *interest* him in it, as often is the case, beyond the painful necessity of memorizing a few distorted notions.

However the primary object of the book is to be used in everyday practice; the writer, being but an average engineer, uses this method to great advantage in working out various problems of construction, etc. Why not suppose that others might likewise derive some benefit from this brief exposition of its principles? Those who want to know more are referred to *Appell,* Mécanique Rationnelle, Vol. II, and *Routh,* Dynamics of Rigid Bodies, Vols. I and II.

My thanks are due to Dr. Eric Doolittle, Director of the Flower Astronomical Observatory, for reading the MSS. and making many valuable suggestions.

N. W. A.

PHILADELPHIA,
December 5, 1916.

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CHAPTER I.

Brief Synopsis of Certain Principles of Dynamics.

1. Constraints. By constraints special conditions are meant, limiting the motion of a particle in a certain manner, prescribed beforehand. For instance a particle may be free to move *only* along a certain curve (a small ring sliding on a curved wire, a car on the track, etc.); or, again, the particle may be compelled to remain, at all times, in contact with a certain surface (for instance if connected by means of a rod to a fixed point, about which it can, therefore, move on a sphere). Very often, the particle can move only on the exterior of a certain surface (imagine, for instance, a small particle sliding off a circular log, a well-known problem); or the distance between a particle and a certain fixed point may be prescribed to be equal to or less than a certain value (case of a stone on a string). All these are typical instances of *constrained motion.*

Analytically, the constraints are specified by geometric equations. For instance, the surface on which the particle is compelled to remain is usually given by some such equation as $f(x, y, z) = 0$; the curve, along which the particle can slide, would be given as the intersection of two surfaces such as $f(x, y, z) = 0$, and $F(x, y, z) = 0$.

It is of the utmost importance to note that the constraints may be either permanent or movable; that is, changing their position or even their shape. Consider, for instance, the motion of a particle constrained to move in a plane which itself is rotating, say, about a vertical axis with a certain angular velocity; or, the motion of a small particle of dust upon a soap bubble while it is being inflated. Conditions of this sort are characterized by the fact that the equations of constraints contain the *time;* that is, the equation of a surface would then be $f(x, y, z, t) = 0$; and the constraining curve would be given by such equations as $f(x, y, z, t) = 0$; and $F(x, y, z, t) = 0$; so that the derivative with respect to time would then not equal 0. If it is 0, this means that the constraints are permanent, or independent of time.

In the absence of constraining conditions the motion of a particle is termed *free.*

2. Virtual work. The fundamental conception of virtual work and virtual velocity is known from elementary treatises (Bowser, Anal. Mech., p. 166). By virtual displacement we shall understand a very small displacement of a particle, *conceived* or *imagined* by us to take place in any direction whatsoever; it may or may not coincide with the displacement actually taking place under the action of the given forces and other conditions; the latter is called *actual* displacement. In case of constrained motion, certain displacements, called *compatible* or *consistent* with the constraints, can be conceived. For instance, in the case of a constraining curve the only compatible displacement would be either backward or forward, from some initial position, along the curve; in the case of a constraining surface, compatible displacements of a particle can be imagined to take place in a great variety of manners, but always subject to the initial condition, viz., adhesion to the surface. Other displacements cannot even be conceived without calling into play the idea of *distorting* the constraints; they are called *inconsistent* with the constraints and will not here be considered; while under free or unconstrained motion the virtual displacements may be any.

For the sake of clearness let us write down the few fundamental principles and definitions established so far: (*a*) By virtual work of a force is meant the product of the virtual displacement of its point of application into the projection of the force upon the direction of the displacement, in other words the virtual work $= P \cdot \delta p \cdot \cos (P, \delta p)$; (*b*) The virtual work of a force for any displacement is equal to the sum of virtual works done by its components; in other words,

$$R \cdot \delta s \cdot \cos (R, \delta s) = \Sigma P \cdot \delta p \cdot \cos (P, \delta p);$$

(*c*) For concurring forces the sum of virtual works done by the forces equals the virtual work done by their resultant; from this is derived the very important form in which virtual work is given in rectangular coordinates (x, y, z). Supposing that there is a force P referred to rectangular axes x, y, z and that the projections of the force upon these axes are (say) X_0, Y_0 and Z_0. Let the virtual displacement of the force be δp, of which the projections upon the axes will be δx, δy, δz. Now in view of what has just been said, the virtual work of the force must equal the sum of virtual works of its components; that is,

$$P \cdot \delta p \cdot \cos (P, \delta p) = X_0 \delta x + Y_0 \delta y + Z_0 \delta z;$$

we will represent this simply by δW; (*d*) From (*c*) it also follows that when any number of concurring forces are in equilibrium the sum of their virtual works is $= 0$. (*e*) To the above the following principle should be added: In case of rotation the virtual work is the product of the moment of the force about the axis of rotation by the angular (virtual) displacement.

All of this refers to free motion. So far as *constrained* motion is concerned the following remarks may be made: If the motion of a particle is constrained, this of course means that at any time the coordinates of the particle must satisfy the constraining equation

$$f(x, y, z) = 0; \qquad \text{or,} \qquad f(x, y, z, t) = 0, \tag{1}$$

otherwise the particle would not remain on the constraining curve or surface; and if a small virtual displacement, of which the projections upon the axes are δx, δy, δz, be given to the particle, compatible with the constraints, then the new posi-