

**AN ELEMENTARY
COURSE IN DESCRIPTIVE
GEOOMETRY**

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An Elementary Course in Descriptive Geoemetry by Solomon Woolf

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SOLOMON WOOLF

**AN ELEMENTARY
COURSE IN DESCRIPTIVE
GEOOMETRY**

AN
ELEMENTARY COURSE
IN
DESCRIPTIVE GEOMETRY.

BY
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