

**WENTWORTH-SMITH
MATHEMATICAL
SERIES; ELEMENTS OF
PROJECTIVE GEOMETRY**

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**GEORGE HERBERT LING & GEORGE
WENTWORTH & DAVID EUGENE SMITH**

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WENTWORTH-SMITH MATHEMATICAL SERIES

ELEMENTS OF
PROJECTIVE GEOMETRY

BY
GEORGE HERBERT LING
GEORGE WENTWORTH
AND
DAVID EUGENE SMITH



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PREFACE

This work has been prepared for the purpose of providing a thoroughly usable textbook in projective geometry. It is not intended to be an elaborate scientific treatise on the subject, unfitted to classroom use; neither has it been prepared for the purpose of setting forth any special method of treatment; it aims at presenting the leading facts of the subject clearly, succinctly, and with the hope of furnishing to college students an interesting approach to this very attractive and important branch of mathematics.

There are at least three classes of students for whom a study of the subject is unquestionably desirable; namely, those who expect to proceed to the domain of higher mathematics, those who are intending to take degrees in engineering, and those who look forward to teaching in the secondary schools. Although the value of the subject to the second of these classes has not as yet been duly recognized in America, European teachers for several decades have realized its usefulness as a theoretical basis for some of the practical work in this field. For the large number of students belonging to the third class, trigonometry, analytic geometry, and projective geometry are the three subjects essential to a fair knowledge of elementary geometry, and it is believed that the presentation given in this book is such as greatly to aid the future teacher. There is a healthy and growing feeling in America that teachers of secondary mathematics need a more thorough training in the subject matter, even at the expense of some of the theory of education which they now have. This being the case, one of the best fields for their study is projective geometry.

It is recognized that students of projective geometry have usually completed an elementary course in analytic geometry and the calculus, that they have a taste for mathematics which leads them to elect this branch of the science, and that therefore there may fittingly be some departure from the elementary methods employed in the earlier mathematical subjects. On the other hand, for some students at least, projective geometry is a transition stage to higher mathematics, and the subject should therefore be presented with due attention to the important and recognized principles which must always be followed in the preparation of a usable textbook.

It is the belief of the authors that they have followed these principles in such a way as to afford to college students a simple but sufficient introduction to this interesting and valuable branch of geometry. Especial attention has been given to the proper paging of the book, to a clear presentation of the great basal propositions, to the illustrations accompanying the text, to the number and careful grading of the exercises, and to the application of projective geometry to the more elementary field of ordinary Euclidean geometry.

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GREEK ALPHABET

The use of letters to represent both numbers and geometric magnitudes has become so extensive in mathematics that it is convenient for certain purposes to employ the letters of the Greek alphabet. In projective geometry the Greek letters are used particularly to represent planes and angles. These letters with their names are as follows:

A	α	alpha	N	ν	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	\omicron	omicron
Δ	δ	delta	Π	π	pi
E	ϵ	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ, ς	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	Y	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

ELEMENTS OF PROJECTIVE GEOMETRY

PART I. GENERAL THEORY

CHAPTER I

INTRODUCTION

1. Orthogonal Projection. In elementary geometry the projection of a point upon a line or upon a plane is usually defined as the foot of the perpendicular from the point to the line or to the plane, and the projection of a line is defined as the line determined by the projections of all its points. This simple projection is called *orthogonal projection*.

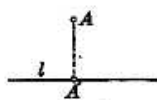


FIG. 1

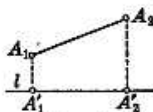


FIG. 2

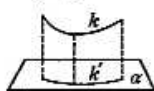


FIG. 3

Thus, the point A' in Fig. 1 represents the orthogonal projection of the point A upon the line l ; the line $A'_1A'_2$ in Fig. 2 represents the projection of the line A_1A_2 upon the line l ; and the line k' in Fig. 3 represents the projection of the curve k upon the plane α .

2. Symbols. Projective geometry, like other branches of mathematics, employs special symbols, generally using capital letters to denote points, small letters to denote lines, the first letters of the Greek alphabet, $\alpha, \beta, \gamma, \delta, \dots$, to denote planes, and the Greek letters ϕ and θ to denote angles.