

**DYNAMICS OF ROTATION; AN  
ELEMENTARY INTRODUCTION  
TO RIGID DYNAMICS. THIRD  
EDITION, REVISED**

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Dynamics of Rotation; An Elementary Introduction to Rigid Dynamics. Third Edition, Revised  
by A. M. Worthington

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**A. M. WORTHINGTON**

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Phys.  
Mech.

# DYNAMICS OF ROTATION

AN ELEMENTARY INTRODUCTION  
TO RIGID DYNAMICS

BY

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## PREFACE TO THE FIRST EDITION

MANY students of Physics or Engineering, who from want either of mathematical aptitude, or of sufficient training in the methods of analytical solid geometry, are unable to follow the works of mathematical writers on Rigid Dynamics, must have felt disappointed, after mastering so much of the Dynamics of a Particle as is given in the excellent and widely-used text-books of Loney, or Garnett, or Lock, to find that they have been obliged, after all, to stop short of the point at which their knowledge could be of appreciable practical use to them, and that the explanation of any of the phenomena exhibited by rotating or oscillating rigid bodies, so interesting and obviously important, was still beyond their reach.

The aim of this little book is to help such students to make the most of what they have already learnt, and to carry their instruction to the point of practical utility.

As a matter of fact, any one who is interested and observant in mechanical matters, and who has mastered the relations between force, mass, and acceleration of velocity of translation, will find no difficulty in apprehending the corresponding relations between couples, moments of inertia, and angular accelerations, in a rigid

body rotating about a fixed axle, or in understanding the principle of the Conservation of Angular Momentum.

Instead of following the usual course of first developing the laws of the subject as mathematical consequences of D'Alembert's Principle, or the extended interpretation of Newton's Second and Third Laws of Motion, and then appealing to the experimental phenomena for verification, I have adopted the opposite plan, and have endeavoured, by reference to the simplest experiments that I could think of, to secure that the student shall at each point gain his *first* ideas of the dynamical relations from the phenomena themselves, rather than from mathematical expressions, being myself convinced, not only that this is the best way of bringing the subject vividly and without vagueness before the learner, but that such a course may be strongly defended on other grounds.

These considerations have determined the arrangement of the chapters and the limitations of the work, which makes no pretence at being a complete or advanced treatise.

My best thanks are due to those friends and pupils who have assisted me in the revision of the proof-sheets and in the working of examples, but especially to my colleague, Mr. W. Larden, for very many valuable suggestions and corrections.

A. M. W.

DEVONPORT, 31st Oct. 1891.

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