

NOTES ON THE GEOMETRY OF THE PLANE TRIANGLE

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649435265

Notes on the Geometry of the Plane Triangle by John Griffiths

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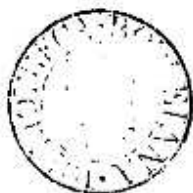
JOHN GRIFFITHS

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GEOMETRY OF THE PLANE
TRIANGLE.

BY

JOHN GRIFFITHS, M.A.,
MATHEMATICAL LECTURER OF JESU'S COLLEGE, OXFORD.



Oxford and London:
JAMES PARKER AND CO.
1867.

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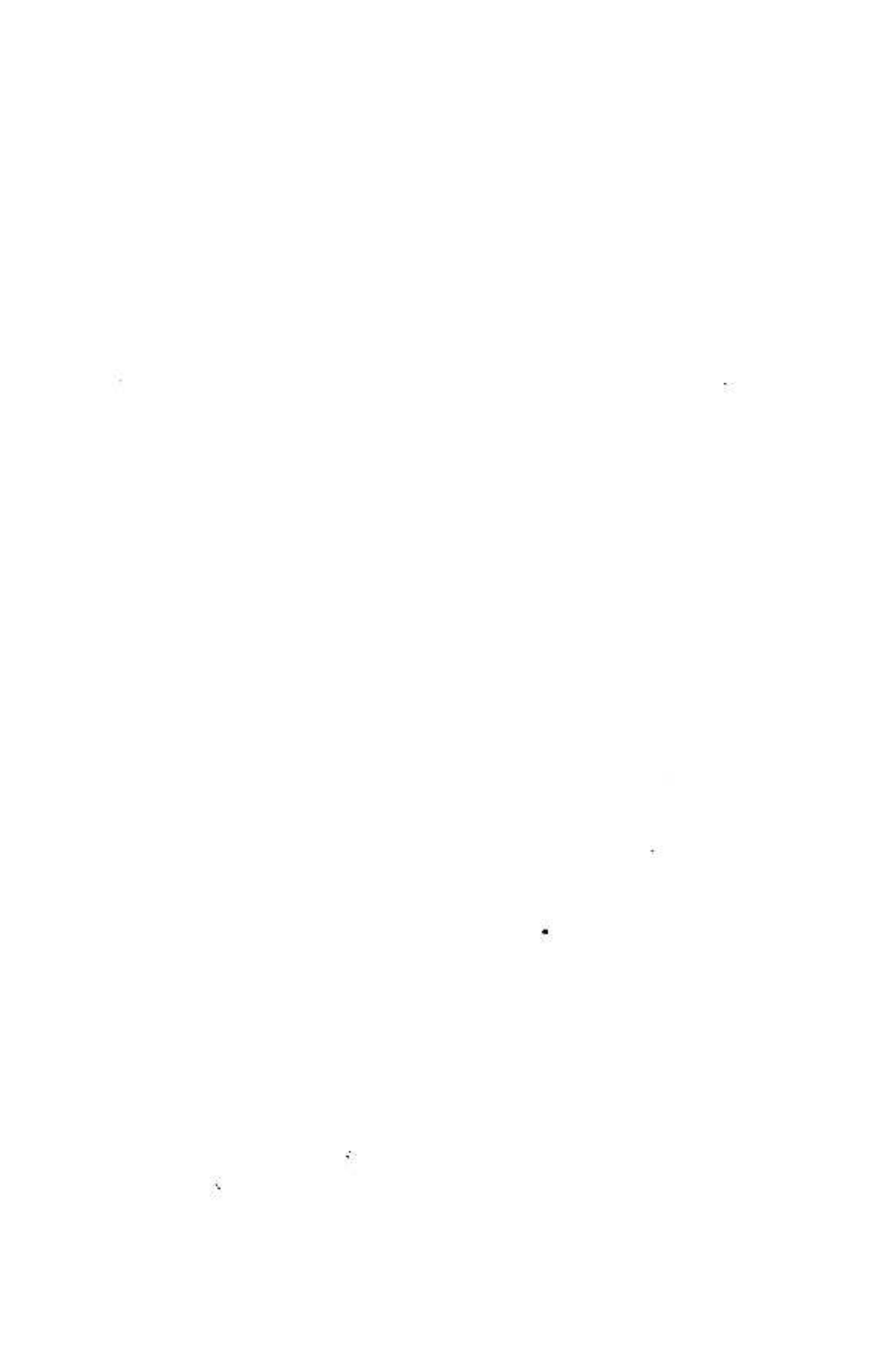


PREFACE.

SOME of the following papers have already appeared in various *Mathematical Journals*. These are now collected, and others are added. They are published, not as a complete treatise on the Geometry of the Triangle, but in the hope that they may be useful by way of reference.

The method adopted is the *Trilinear*, though in some cases *Cartesian co-ordinates* have been employed. The author would be glad if this slight sketch should induce some competent *Mathematician* to prosecute the subject further, and to write an exhaustive treatise on the properties of the Triangle.

JESUS COLLEGE, OXFORD,
June 1, 1867.



CONTENTS.

I.—VII. Properties of the polar circle.

VIII. The three pairs of points common to the circles (AA_1) , (BC) ; (BB_1) , (CA) ; (CC_1) , (AB) lie on the same circle. See the Notation, p. viii.

Expressions found for the centre and radius of the circle

$$2a \cos Aa^2 + b^2 \cos Bb^2 + c^2 \cos Cc^2.$$

IX. The three pairs of points common to the circles (AA_1) , (PBC) ; (BB_1) , (PCA) ; (CC_1) , (PAB) lie on the same circle.

X. The circles (ABC) , $(A_1B_1C_1)$, and (PG) are co-axial; the latter being shewn to be the locus of a point at which the first two appear of equal size.

Consequences.

XI. If on the segments PA , PB , PC , three points p , q , r be taken such that

$$Pp = \frac{1}{n} \cdot PA; Pq = \frac{1}{n} \cdot PB; Pr = \frac{1}{n} \cdot PC; \text{ and on } PA_1, PB_1, PC_1$$

$$\text{three other points } p', q', r' \text{ such that } Pp' = \frac{2}{n} \cdot PA_1; Pq' = \frac{2}{n} \cdot PB_1;$$

$$Pr' = \frac{2}{n} \cdot PC_1; \text{ then } p, p', q, q', r, r' \text{ are concyclic.}$$

Condition 1. that the circle $(pp'qq'r'r')$ shall touch the inscribed circle.

2. That it shall be orthogonal to the polar circle.

XII.—XVI. Properties of the nine-point circle.

XVII. The feet of the perpendiculars let fall from (ξ, η, ζ) , $\left(\frac{1}{\xi}, \frac{1}{\eta}, \frac{1}{\zeta}\right)$ upon the sides of the triangle of reference are concyclic.

Equation of the inscribed conic whose foci coincide with the pair of inverse

$$\text{points } (\xi, \eta, \zeta); \left(\frac{1}{\xi}, \frac{1}{\eta}, \frac{1}{\zeta}\right).$$

Equation to the radical axis of the "Pedal" of (ξ, η, ζ) and the circumscribing circle $\Sigma a\beta\gamma$; condition that they shall touch.

Equations to certain loci.

XVIII. The pedal of (ξ, η, ζ) ; the circle $\Sigma a\beta\gamma$, and the director of the conic

$$\Sigma \sqrt{\frac{\xi(\eta + \zeta \cos A)(\eta \cos A + \zeta)}{\cos A}} \cdot a, \text{ are co-axal.}$$

Deduction.

XIX. The pedals of (ξ, η, ζ) , (ξ', η', ζ') , and the director of the inscribed conic

$$\Sigma \sqrt{\left| \begin{array}{cc} \eta & \zeta \\ \eta' & \zeta' \end{array} \right| \left| \begin{array}{cc} \eta & \zeta' \\ \zeta & \eta' \end{array} \right| \xi \xi'} \cdot a = 0, \text{ are co-axal.}$$

Seven-tangent conic.

Consequences, 1^o—7^o.

1^o. The directors of the four conics which have a, β, γ for tangents, and pass through O and P , $(\cos A, \cos B, \cos C)$; $(\sec A, \sec B, \sec C)$ touch the nine-point circle at its points of contact with the inscribed and escribed circles.

XX. The inscribed and escribed circles I and E_o , and the pedal of (ξ, η, ζ) are co-axal.

XXI. Condition that the pedal of (ξ, η, ζ) shall touch the inscribed circle I .

XXII. The circumscribing (ABC) , the nine-point circle (A_1, B_1, C_1) , and the polar circle are co-axal.

Consequences.

XXIII. If D', E', F' denote the vertices of the triangle formed by the tangents at A, B, C to the circle (ABC) ; then the circles (ABC) , (DEF) and $(D'E'F')$ are co-axal.

Consequences.

The two points where the circumscribed circle (ABC) is met by any one of the escribed circles shown to be the foci of the two parabolas which pass through A, B, C and h_1 ; h_2 being the centre of homology of ABC and the triangle formed by joining the points where the escribed circle in question touches the sides.

Application to the nine-point circle.

XXIV., XXV. Other properties of the triangles $D'E'F'$.

Consequences.

Résumé. System of co-axial circles.

XXVI. The director of any conic which touches α, β, γ , and passes through O , the centre of the circumscribed circle, has contacts of a similar species with this circle and the nine-point circle.

Consequences.

XXVII. Equation of the locus of a point (ξ, η, ζ) such that its pedal circle touches the nine-point circle.

Feuerbach's theorem.

XXVIII. Given five concyclic points, the centres of the five equilateral hyperbolas which pass through them, taken four together, are also concyclic.

Consequences.

XXIX. Given four concyclic points, the centres of the nine-point circles of the four triangles that can be formed from them are also concyclic.

Consequences.

XXX. Given four concyclic points, the centroids of the four triangles formed by them are also concyclic.

Property of the "Centroid-circles" of the five quadrangles formed by five concyclic points,

Theorem by Mr. Townsend.

XXXI. Extension of the theorem of Sect. XVII.

APPENDIX. Note on the theorem of Sect. XXVIII.