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OSCAR S. ADAMS

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U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, SUPERINTENDENT

CARTOGRAPHY

LAMBERT PROJECTION

Tables for the United States

BY

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Geodetic Computer
United States Coast and Geodetic Survey

Special Publication No. 52

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E R R A T A

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PREFACE.

This publication consists of a projection table for the United States computed upon the Lambert conformal conic projection with two standard parallels. The term "conformal" used in connection with a projection means that any very small figure upon the earth is represented by a similar small figure upon the map. This entails two qualities. For short distances at any place the scale is the same in all directions, and the angle between two curves upon the earth is preserved in their representations upon the map. In consequence of the second, the azimuths of lines are the same upon the map as upon the earth.

The tables were computed in meters and the resulting values reduced to yards. The meter tables are given in the first part of this book and are followed by the tables in yards. Each table consists of three parts. In the first part, the coordinates are referred to an arbitrary origin lying in the Pacific Ocean somewhere off the west coast of Mexico. The second part is the same general table referred to the 96° meridian as Y axis and a perpendicular to the same at its intersection with the 39° parallel as X axis. The third part consists of a projection table for local maps upon the Lambert system.

In the first part of the text is found a mathematical development of the Lambert projection, which is followed by the computation of the elements of the projection for the United States, including also examples of the general computation. One desiring only information which will enable him to construct a projection, without having to study and learn the theory on which the projection is based, may confine himself to pages 12 to 27. Here is given a detailed exposition of the method of constructing a projection by means of the tables. The first part of this descriptive matter refers to the use of the table with the arbitrary origin. Next is given the description of the use of the table with the origin at the approximate center of the United States. The last part contains a full account of the construction and use of the table for local maps. In each of the three parts the construction is fully illustrated by diagrams. A solution of the various questions that may arise in the application of the system is given on pages 27 to 30. The closing paragraphs, pages 31 and 32, are devoted to a discussion of the advantages and limitations of the use of the Lambert system of projection.

Much credit is due to the members of the divison of Geodesy for their hearty cooperation in the rapid computation of the tables and to several members of the division of charts for their assistance in designing the illustrations that will materially aid in the use of the tables for the construction of projections.

A more detailed statement of some of the questions connected with the Lambert projection may be found in Special Publication No. 47, by C. H. Deetz, cartographer, of the Coast and Geodetic Survey.

LAMBERT PROJECTION

TABLES FOR THE UNITED STATES.

By Oscar S. Adams,

Geodetic Computer, U. S. Coast and Geodetic Survey.

MATHEMATICAL DEVELOPMENT.

It is proposed to determine a projection of the spheroid with the parallels as concentric circles and the meridians as radii of this system of circles such that the small figure $ABCD$ shall be exactly similar to the small figure upon the earth that it represents. Let a be the semimajor axis and b the semiminor axis of the earth; $\epsilon=\sqrt{\frac{a^2-b^2}{a^2}}$, the eccentricity; λ the longitude, ϕ the latitude, and p the colatitude. It is evident that the angle AOB, figure 1, will be proportional to the difference of longitude between A and B, denoted by $d\lambda$. l is an arbitrary constant of proportionality that is generally taken less than unity. Let $OB=r$; then $BD=dr$, and $AB=rld\lambda$. But the length of BD upon the earth is equal to $\frac{a(1-\epsilon^2)\,dp}{(1-\epsilon^2\cos^2 p)^{3/2}}$ and the length of AB upon the earth is equal to $\frac{a\,d\lambda\sin p}{(1-\epsilon^2\cos^2 p)^{1/2}}$.

Fig. 1.

If the desired similarity is to be attained, the following proportion should be valid:

$$dr : lrd\lambda = \frac{a(1-\epsilon^2)\,dp}{(1-\epsilon^2\cos^2 p)^{3/2}} : \frac{a\,d\lambda\sin p}{(1-\epsilon^2\cos^2 p)^{1/2}}$$

or,

$$\frac{dr}{lr}=\frac{(1-\epsilon^2)\,dp}{(1-\epsilon^2\cos^2 p)\sin p}$$

$$\frac{dr}{lr}=\frac{dp}{\sin p}-\frac{\epsilon^2\sin p\,dp}{2(1+\epsilon\cos p)}-\frac{\epsilon^2\sin p\,dp}{2(1-\epsilon\cos p)}$$

$$\frac{1}{l}\int\frac{dr}{r}=\int\frac{dp}{\sin p}+\frac{\epsilon}{2}\int\frac{-\epsilon\sin p\,dp}{1+\epsilon\cos p}-\frac{\epsilon}{2}\int\frac{\epsilon\sin p\,dp}{1-\epsilon\cos p}$$

$$\frac{1}{l}\log r-\frac{1}{l}\log K=\log\tan\frac{p}{2}+\frac{\epsilon}{2}\log\frac{1+\epsilon\cos p}{1-\epsilon\cos p}$$

$\frac{1}{l}\log K$ being the constant of integration depending upon the limits chosen and hence for the present an arbitrary constant.

$$\log\frac{r}{K}=l\log\left[\tan\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\frac{\epsilon}{2}}\right]$$

or

$$r=K\tan^{l}\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\frac{l\epsilon}{2}}.$$

It is evident that any small figure upon the earth will be represented by a similar small figure upon the map and that the projection will be conformal.

If an angle z is assumed such that

$$\tan\frac{z}{2}=\tan\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\frac{\epsilon}{2}}$$

this angle will be very nearly equal to the complement of the geocentric latitude. If ϕ' is the geocentric latitude, the relation between ϕ and ϕ' is

$$\tan\phi'=\frac{b^2}{a^2}\tan\phi.$$

Then to a sufficient degree of approximation

$$z=\frac{\pi}{2}-\phi'.$$

The value of z can be computed rigidly very conveniently by assuming an angle q such that

$$\cos q=\epsilon\cos p$$

then

$$\cot\frac{q}{2}=\sqrt{\frac{1+\cos q}{1-\cos q}}$$

so that

$$\tan\frac{z}{2}=\tan\frac{p}{2}\cot^{\epsilon}\frac{q}{2}$$

or

$$\log\tan\frac{z}{2}=\log\tan\frac{p}{2}+\epsilon\log\cot\frac{q}{2}.$$

However, the approximate formula determines z to within a few tenths of a second.

With this angle the formula for r becomes

$$r = K \tan^l \frac{z}{2}.$$

The value of l may be determined so as to hold the ratio of equal arcs of any two chosen parallels. If N is the length of the normal prolonged to the minor axis, the length of a radian of the parallel of latitude ϕ_1 is represented by $N_1 \cos \phi_1$; similarly, the length of a radian of parallel of latitude ϕ_2 is given by $N_2 \cos \phi_2$. The ratio of the two arcs is represented by $\frac{N_1 \cos \phi_1}{N_2 \cos \phi_2}$.

Since the A factor in the tables for the computation of geodetic positions[1] is defined by the equation

$$A = \frac{1}{N \sin 1''}$$

this ratio becomes

$$\frac{A_2 \cos \phi_1}{A_1 \cos \phi_2}.$$

The length of the arc upon the map that represents this radian of the parallel ϕ_1 is equal to

$$lr_1 = lK \tan^l \frac{z_1}{2};$$

likewise that of parallel ϕ_2 is represented by

$$lr_2 = lK \tan^l \frac{z_2}{2}.$$

To preserve the ratio required, the following proportion must be true:

$$\left(\frac{\tan \frac{z_1}{2}}{\tan \frac{z_2}{2}} \right)^l = \frac{A_2 \cos \phi_1}{A_1 \cos \phi_2}$$

or

$$l = \frac{\log \cos \phi_1 - \log \cos \phi_2 - \log A_1 + \log A_2}{\log \tan \frac{z_1}{2} - \log \tan \frac{z_2}{2}}.$$

[1] Special publication No. 8, U. S. Coast and Geodetic Survey.