

**A TREATISE ON PLANE  
AND SPHERICAL  
TRIGONOMETRY**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649184262

A treatise on plane and spherical trigonometry by William Chauvenet

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**WILLIAM CHAUVENET**

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A  
TREATISE  
ON  
PLANE AND SPHERICAL  
TRIGONOMETRY.

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*Fifth Edition.*

PHILADELPHIA:  
J. B. LIPPINCOTT & CO.  
1871.

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Entered according to Act of Congress, in the year 1860, by WILLIAM CHAUVENET, in the  
Clerk's Office of the District Court of the Eastern District of Pennsylvania.

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## PREFACE.

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I HAVE in this treatise endeavored to arrange a course of trigonometrical study sufficiently extensive to enable the student to comprehend readily any applications of trigonometry he may meet with in the works of the best modern mathematicians. With this object, some topics have been introduced which are not usually found in works devoted specially to this subject.

Among those topics, the most important is the solution of the general spherical triangle, or the triangle whose sides and angles are not limited, according to the usual practice, to values less than  $180^\circ$ . The advantage of introducing such triangles into astronomical investigations is sufficiently shown in the applications made of them in the works of BESSEL and other German mathematicians; and especially in the *Theoria Motus Corporum Cœlestium* of GAUSS, who was the first to suggest their employment.

The subject of Finite Differences of triangles, plane and spherical, occupies a large space in Cagnoli's treatise, but has not been admitted into more recent works. It here occupies only a few pages, but no important result of

Cagnoli's Table has been omitted, while a number of the formulæ are much simpler than the corresponding ones given by him.

Although my plan embraces a much more extensive course than is contained in the text-books commonly used, I have studiously kept in view the wants of academic and collegiate classes; and have so arranged the work that a selection of subjects of immediate importance may be readily made. The more elementary portions are printed in a larger type, and are intended to form, independently of the matter in the smaller type, a connected treatise which may be studied as though it were in a separate volume.

Those who may afterwards wish to extend their knowledge will appreciate the advantage of having the higher departments of the subject treated in connection with those fundamental ones to which they are most intimately related.

W. C.

U. S. NAVAL ACADEMY,  
*Annapolis, Md., May 1, 1860*

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#### NOTE TO THE FOURTH EDITION.

In this edition, besides a number of minor changes, and the correction of some typographical errors, a very important modification has been made in the solution of the equation  $\tan x = p \tan y$  by series (p. 145), which was given in former editions in the usual form as stated by all writers on trigonometry. This form was discovered to lack generality, and consequently to fail in certain applications, in consequence of the omission of the arbitrary term  $n\pi$  now introduced. Several subsequent investigations, depending on this, have in like manner been rectified.

W. C.

U. S. NAVAL ACADEMY, *April 1, 1864.*



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# CONTENTS.

## PART I. PLANE TRIGONOMETRY.

|  | PAGE |
|--|------|
| CHAPTER I.   |      |
| MEASURES OF ANGLES AND ARCS.....                               | 9    |
| CHAPTER II.  |      |
| SINES, TANGENTS, AND SECANTS. FUNDAMENTAL FORMULE.....         | 14   |
| CHAPTER III.   |      |
| TRIGONOMETRIC FUNCTIONS OF ANGULAR MAGNITUDE IN GENERAL.....   | 22   |
| Sine and Tangent of a Small Angle or Arc.....                  | 30   |
| CHAPTER IV.  |      |
| GENERAL FORMULE.....   | 31   |
| Formule for Multiple Angles.....                               | 36   |
| Relations of Three Angles.....                                 | 38   |
| Inverse Trigonometric Functions.....                           | 41   |
| CHAPTER V.   |      |
| TRIGONOMETRIC TABLES.....                                      | 43   |
| Elementary Method of Constructing the Trigonometric Table..... | 47   |
| CHAPTER VI.  |      |
| SOLUTION OF PLANE RIGHT TRIANGLES.....                         | 51   |
| Additional Formule for Right Triangles.....                    | 54   |
| CHAPTER VII.   |      |
| FORMULE FOR THE SOLUTION OF PLANE OBLIQUE TRIANGLES.....       | 57   |

|   | PAGE |
|---|------|
| CHAPTER VIII.   |      |
| SOLUTION OF PLANE OBLIQUE TRIANGLES.....  | 61   |
| Area of a Plane Triangle.....   | 74   |
| CHAPTER IX.   |      |
| MISCELLANEOUS PROBLEMS RELATING TO PLANE TRIANGLES.....   | 75   |
| CHAPTER X.  |      |
| SOLUTION OF CERTAIN TRIGONOMETRIC EQUATIONS AND OF NUMERICAL EQUATIONS OF THE SECOND AND THIRD DEGREE.....  | 85   |
| CHAPTER XI.   |      |
| DIFFERENCES AND DIFFERENTIALS OF THE TRIGONOMETRIC FUNCTIONS.....   | 101  |
| CHAPTER XII.  |      |
| DIFFERENCES AND DIFFERENTIALS OF PLANE TRIANGLES.....   | 105  |
| CHAPTER XIII.   |      |
| TRIGONOMETRIC SERIES. DEVELOPMENTS OF THE FUNCTIONS OF AN ANGLE IN TERMS OF THE ARC, AND RECIPROCALLY.....  | 115  |
| Computation of Natural Sines and Cosines by Series.....   | 116  |
| Computation of the Ratio of the Circumference of a Circle to its Diameter   | 120  |
| Computation of Logarithmic Sines and Cosines.....   | 122  |
| CHAPTER XIV.  |      |
| EXPONENTIAL FORMULÆ. TRINOMIAL OR QUADRATIC FACTORS.....  | 127  |
| CHAPTER XV.   |      |
| TRIGONOMETRIC SERIES CONTINUED. MULTIPLE ANGLES.....  | 135  |
| Development of the Sine and Cosine of the Multiple Angle in a Series of Ascending Powers of the Cosine of the Simple Angle.....                                       | 137  |
| Development of the Sine and Cosine of the Multiple Angle in a Series of Ascending Powers of the Sine of the Simple Angle.....   | 139  |
| Development of the Sine and Cosine of the Multiple Angle in a Series of Ascending Powers of the Tangent of the Simple Angle.....                                      | 140  |
| Development of any power of the Cosine of the Simple Angle in a Series of Sines or Cosines of the Multiple Angles, the Cosine of the Simple Angle being positive..... | 141  |
| Development of any power of the Cosine of the Simple Angle in a Series of Sines or Cosines of the Multiple Angles, the Cosine of the Simple Angle being negative..... | 142  |
| Development of any power of the Sine of the Simple Angle in a Series of Sines or Cosines of the Multiple Angles.....  | 144  |
| Certain Equations developed in Series of Multiple Angles.....   | 145  |

## PART II.

## SPHERICAL TRIGONOMETRY.

| CHAPTER I.  |  | PAGE |
|---|--|------|
| GENERAL FORMULÆ.....  |  | 149  |
| Gauss's Theorem.....  |  | 161  |
| Additional Formulæ.....   |  | 162  |
| Deduction of the Formulæ of Plane Triangles from those of Spherical<br>Triangles..... |  | 166  |
| CHAPTER II.   |  |      |
| SOLUTION OF SPHERICAL RIGHT TRIANGLES.....  |  | 167  |
| Additional Formulæ for the Solution of Spherical Right Triangles.....                 |  | 176  |
| Quadrantal and Isosceles Triangles.....   |  | 177  |
| CHAPTER III.  |  |      |
| SOLUTION OF SPHERICAL OBLIQUE TRIANGLES.....  |  | 178  |
| Solution of Spherical Oblique Triangles by means of a Perpendicular.....              |  | 206  |
| Computation of Spherical Formulæ by the Gaussian Table.....                           |  | 211  |
| CHAPTER IV.   |  |      |
| SOLUTION OF THE GENERAL SPHERICAL TRIANGLE.....                                       |  | 214  |
| Note upon Gauss's Equations.....  |  | 227  |
| CHAPTER V.  |  |      |
| AREA OF A SPHERICAL TRIANGLE.....   |  | 229  |
| CHAPTER VI.   |  |      |
| DIFFERENCES AND DIFFERENTIALS OF SPHERICAL TRIANGLES.....                             |  | 232  |
| CHAPTER VII.  |  |      |
| APPROXIMATE SOLUTION OF SPHERICAL TRIANGLES IN CERTAIN CASES.....                     |  | 241  |
| Legendre's Theorem.....   |  | 244  |
| CHAPTER VIII.   |  |      |
| MISCELLANEOUS PROBLEMS OF SPHERICAL TRIGONOMETRY.....                                 |  | 246  |