# MATHEMATICAL QUESTIONS WITH THEIR SOLUTIONS, FROM THE "EDUCATIONAL TIMES," WITH MANY PAPERS AND SOLUTIONS NOT PUBLISHED IN THE "EDUCATIONAL TIMES". VOL. XXXIII

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Mathematical Questions with Their Solutions, from The "Educational Times," with Many Papers and Solutions Not Published in The "Educational Times". Vol. XXXIII by W. J. C. Miller

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# W. J. C. MILLER

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# MATHEMATICAL QUESTIONS

# SOLUTIONS,

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FROM THE "EDUCATIONAL TIMES,"

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EDITED BY

W. J. C. MILLER, B.A., REGISTRAE OF THE GENEBAL MEDICAL COUNCIL.

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### CORRIGENDA.

#### VOL. XXXIII.

Page 60, line 28, for these read there. Page 82, line 5 from bottom, for 2668 read 6268.

#### VOL. XXXII.

Page 72, omit from line  $\delta$  from bottom to line  $\delta$  from bottom of page 73; the paragraph to be left out belonging to the Solution of the closely related Question  $\delta$ 276, by the same author (Vol. XXXI., p. 27.)

• Of this series thirty-three volumes have now been published, each volume containing, in addition to the papers and solutions that have appeared in the *Educational Times*, about the same quantity of new articles, and comprising contributions, in all branches of Mathematics, from most of the leading Mathematicians in this and other countries.

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### Solbed Questions.

1433. (The Editor.)-Prove the following reciproca cases of involution :-

a. The three sides of every triangle, and every three concurrent lines through its three vertices, intersect every axis in six points in involution.

a'. The three vertices of every triangle, and every three collinear points on its three sides, subtend every vertex in six rays in involution.

b. The six perpendiculars on the six lines from any point in the former case determine at the point a pencil of six rays in involution.

b'. The six perpendiculars from the six points upon any line in the latter case determine on the line a system of six points in involution. ..... 50

1588. (Professor Sylvester, F.R.S.)-Solve the equation ax6 + 5bx4 + 10cax8 + 10cbx2 + 5c2ax + c2b = 0. ..... 115

1628. (Professor Sylvester, F.R.S.)—Extending the meaning of the word Bicorn to signify the general curve defined by the equations

$$x = \frac{e^3}{\phi^4 + \phi^3}, \ y = \frac{e^2 (2\phi + 3)}{\phi^3 + \phi^2},$$

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- 2782. (Professor Crofton, F.R.S.)—(1) A given point is known to be within a certain circle of given radius, but unknown position; find the chance that another given point is also within the circle. Also (2) three given points are known to be within a scrtain circle, which is otherwise altogether unknown: determine the most probable position of its centre. Again, (3) two given point is are known to be within a circle, and a third given point is known to be outside it: determine the most probable position of its centre.

- 4549, 4566, 4578, 4579. (C. Lendesdorf, M.A., and E. B. Elliott, M.A.) A bag contains mn balls, each equally likely to be of any one of m different colours. All but m are drawn out at random, and found to be n-1 of each colour. Find the chance that the remaining m are (1) one of each colour, (2) all of one colour.... 68
- 4817. (Professor Sylvester, F.R.S.)—In a school of 16 girls, a rule has been laid down that they shall walk out every day in rows of threes, but that the same two girls shall never come together twice in the same row. The rule is supposed to have been carried out correctly during the six working days of the week,

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but when the time comes for their going to church together on Sunday it is found to be absolutely impossible to continue it any further. Investigate whether the rule can have been correctly carried out during the six previous days ; and if so, show how. 53

- 5113. (The Editer.)-124, 164, 166, 163, 234, 132, 473, 456, 467, 367, and 124, 143, 126, 234, 167, 523, 173, 662, 667, 673 are two sets of ten triplets each made with seven elements, so that every duad employed occurs twice in either system. If that every dual employed occurs twice in etter system. If we change them by any substitution, as 2135746-which means, put 2 for 1, 1 for 2, 3 for 3, 5 for 4, 7 for 5, 4 for 6, and 6 for 7-the first becomes 216, 276, 274, 243, 135, 231, 563, 574, 546, 346, which is similar to the first; but no substitution can make the first into the second, or the second into the first; so that these two are dissimilar sets. Find the entire number of dissimilar sets. ..... 114
- 5138. (The Editor.)— Two persons make an appointment to meet on a certain day at the Medical Council Office, but without fixing the time further than that it is to be between 2 and 3 o'clock; and each agrees to wait ten minutes for the other. Now, supposing all times between the specified limits for coming to the Office to be equally probable, show, by a general solution, that the odds are 25 to 11 against the meeting taking place.....
  - 78
- 5167. (Professor Seitz, M.A.)—A point is taken at random in the surface of a given circle, and from it a line equal in length to the radius is drawn, so as to lie wholly in the surface of the circle; find the chance that the line intersects a given diameter. 52
- 5264. (Rev. H. G. Day, M.A.)-Two similar polygons ABCDE..., A<sub>1</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>E<sub>1</sub>... are similarly situated on lines AB, A<sub>1</sub>B<sub>1</sub> (not parallel); show that (1) if they are situated towards the same parts, a point O can be found at which AA<sub>3</sub>, BB<sub>1</sub>, CC<sub>1</sub>, &c. all subtend the same angle : and (2) if towards opposite parts, a point O can be found at which the bisectors of the angles AOA<sub>1</sub>, BOB, COC, coincide. 43
- 5272. (Professor Townsend, F.R.S.)-A solid of revolution of uniform density, turning without friction round a fixed point on its axis of figure, and rolling without sliding on a fixed plane inclined at any angle to the horizon, being supposed to have the velocity of rotation just sufficient to carry it with exhausted energy of motion to its position of unstable equilibrium against the action of gravity ; determine, in finite terms, its time of passage from the opposite position of stable equilibrium to any other on its way ..... .....
- 5329. (Professor Towasend, F.R.S.)—A rigid body, in unconstrained equilibrium in free space, being supposed set in motion by a single impulsive force applied at a definite point of its mass; if the initial motion be a pure rotation, show that-(a) The several possible lines of impulsive action through the

point generate a quadriccone, containing the three perpendiculary from the point on the three central principal planes, and the connector of the point with the centre of instria of the body. (b) The several corresponding axes of initial rotation in the

body envelope a quadric cone, touching the three central princi-pal planes, and the diametral plane conjugate to the direction of the point with respect to the central ellipsoid of the body....