ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS, PP. 1-237

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649571260

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Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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CALCULUS,

BY

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Second Edition.

PORTLAND,

PUBLISHED BY SANBORN & CARTER.

PRESS OF J. GRIFFIN, BRUNSWICK.

1859.

PREFACE TO THE FIRST EDITION.

The principles of the Calculus are reached from three different points of view in the methods respectively of Leibnitz, Newton, and Lagrange. The method of the latter not being adapted to elementary instruction, the choice for this purpose lies between those of the two former. The recent text-books, both English and French, are in general based upon the method of Newton. The expediency of this may well be questioned. The artifice which lies at the basis of the Calculus, consists in the employment of certain special auxiliary quantities adapted to facilitate the formation of the equations of a problem. The limit, or differential coefficient, the auxiliary employed in the method of Newton, is not easily represented to the mind, and being composed of two parts which cannot be separately considered, it is with the more difficulty applied to the solution of problems. On the other hand the differential, the auxiliary employed in the method of Leibnitz, is simple in itself, is very readily conceived, and adapts itself with wonderful facility to all the different classes of questions which require for their solution the aid of the Calculus. From these and similar considerations, confirmed by the results of experience, I do not hesitate to prefer the method of Leibnitz as the basis of an elementary work on the Calculus. A perfect knowledge of the subject requires, indeed, that it should be examined from all the different points of view which have been named. But the learner should begin with the method of Leibnitz. In accordance with these views the present work has been prepared. The following analysis exhibits the course pursued.

The first object is to show the necessity for the new instrument of investigation we are about to examine. A simple problem is, therefore, introduced for which an equation cannot be directly obtained in terms of the quantities which enter into it, and which require to be considered. In these circumstances the ordinary expedient of analysis, is the employment of auxiliary quantities to aid in the formation of the equation required. But from the variable nature of the quantities which the problem presents, the auxiliaries most suitable to be employed, it is obvious, are not the finite and determinate quantities of ordinary algebra, but certain indeterminate quantities capable of being taken as small as we please, without changing thereby the values of the quantities for which they are to be substituted or in connection with which they are to be used. We are thus led to the consideration of quantities infinitely small, or infinitesimals; which form the subject of the first section.

The auxiliaries employed in any case must, it is evident, sustain relations to their primitives, in virtue of which they may be eliminated, after they have served the purpose for which they are introduced. We have, then, next to establish such relations for the new auxiliaries we wish to employ. The variable quantities of the problem may be regarded as changing their values by increments or decrements infinitely small. If we take, then, the difference between any two successive states of the variables, the results will be quantities infinitely small, which will have also a necessary relation to the primitives from which they are derived. We may employ them, therefore, for the auxiliaries required. They are called differentials; and the process by which they are obtained is called differentiation; the rules for which, for simple algebraic quantities, are developed in the second section.

We have now our instrument in part, and proceed at once to its use. We commence with the problem of tangents, one of the most interesting of the problems which occupied the attention of the ablest geometers of antiquity, and which long defied their efforts and baffled their skill. We find it yield at once to the instrumentality we have now acquired. The third section is occupied with various particular cases of this general problem, and the student is thus introduced to the Differential Calculus, of the nature of which he is now enabled to form a general idea.

In the problems solved thus far, the quantities whose relations are sought enter directly into the differential equations obtained. We are able, therefore, to eliminate by the ordinary processes of algebra, the auxiliaries employed. But we meet next with a class of problems in which this cannot be done. And in which the only mode of freeing the equations from the indeterminates employed consists in returning from them to their primitives, by a process the reverse of that by which they are obtained.

The infinitely small quantities we have employed as auxiliaries, may be regarded as the elements of which the variable quantities of the problem are respectively composed. This reverse process will then consist in the summing up or taking the whole of these elements. It is called integration, the rules for which, for simple algebraic quantities, are developed in the fourth section. Proceeding immediately to the application of our instrument as now improved, we introduce in the fifth section various problems upon the quadrature of areas, and the cubature of solids; the solution of which gives the student an elementary view of the Integral Calculus.

The problems of the rectification of curve lines, and the quadrature of the surfaces of the solids of revolution, requiring the aid of both the processes of elimination already employed, brings into use the entire instrument. It is thus seen as a whole, and the student has now a general idea of THE CALCULUS, as the entire instrument by way of eminence is called. What remains, is only its more full development, with such new applications as will most clearly exhibit its power.

With this object in view, we pass to the sixth section. A proposed problem being too difficult for the auxiliaries already obtained, we naturally seek for additional ones by a repetition of the artifice already employed. Regarding our first auxiliaries, therefore, as now becoming primitives in their turn, we take their differentials, and again the differentials of these last, and so on. We thus obtain second, third, &c. differentials. These form the subject of the sixth section. Making immediate application of these new auxiliaries, we introduce in the seventh section the problem of the development of a function of one or more variables, and obtain the important theorems of Maclaurin and Taylor. And with resources thus increased, we discuss at large in the eighth and ninth sections the great problem of maxima and minima, and the whole theory of curves.

Thus far our problems have involved algebraic quantities only. Passing next to those which involve transcendental functions, we develop in the tenth section rules both for the differentiation and integration of quantities of this description. The discussion of the sinusoid, the logarithmic curve, and the spirals in general, presents in the eleventh section an application of the Calculus to problems depending upon transcendental quantities.

It must, by this time, have been perceived that the Differential Calculus is far in advance of the Integral. The twelfth section is, therefore, occupied with the development of various artifices and methods of reduction adapted to facilitate the process of integration, or the elimination of the auxiliaries we have occasion to employ. With these increased facilities we advance, in the thirteenth section, to a full examination of the nature and properties of the Cycloid, the most beautiful as well as important of all the transcendental curves. Here the full power of the Calculus is more distinctly exhibited. Problems are solved by it, as with the dash of a pen, which eluded the grasp of the most distinguished of the ancient geometers, or were solved only after laborious efforts, and by methods alike circuitous and complicated.

In the fourteenth section, some additional problems upon the quadrature of areas and cubature of solids are introduced, and with these is closed the application of the Calculus to problems of pure geometry.

In the fifteenth and sixteenth sections we enter a wider field and upon topics of a higher interest. In these the Calculus is applied to various problems of Mechanics involving varied motion, motion along curve lines, and the determination of forces; problems upon equilibrium, the determination of the centre of gravity of bodies, and the pressure and discharge of fluids. We thus see the application of the Calculus to the Physical Sciences, the sphere to which it is specially adapted, and within which its powers have been most remarkably displayed.

At this point, we take our final step in the improvement of the instrument we are considering. We come to a class of problems far transcending in difficulty those which have previously been solved; and which require for their solution a higher degree of indeterminateness in the auxiliaries employed. The new class of auxiliaries required are differentials under a new point of view. They are called variations. Their derivation with some examples of their use forms the subject of the seventeenth section. We close our necessarily limited view of the sublime instrument before us, by its application in the eighteenth section to two of the most important problems of astronomy—the attraction of spheres, and the investigation of the law of the force which binds the planets and comets to their orbits. The work terminates in the nineteenth section, with a brief exposition of the methods of Newton and Lagrange, and a few miscellaneous examples.

The plan of the work differs, it will be perceived, widely from the many excellent text-books on the subject recently published. It is submitted with diffidence in regard to its execution, but with great confidence that the plan itself is well adapted to introduce the student by a natural and easy process to a general knowledge of the Calculus, to enable him clearly to understand in what it consists, and to discern its power; and especially to awaken in him an interest in those profound investigations in respect to which it is the appropriate instrument.

The work completes the course of text-books of pure mathematics prepared by the author. Its materials have been selected from the ordinary sources. The general view taken of the philosophy of the Calculus is the same with that of Carnot in his Reflections on the Metaphysics of the Differential and Integral Calculus, and of Comte in his Positive Philosophy.

WM. SMYTH.

Bowd. Coll., March, 1854.