ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS. METHOD OF RATES

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ARTHUR SHERBURNE HARDY

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METHOD OF RATES.

BY

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PREFACE.

This text-book is based on the method of rates, which, in the experience of the author, has proved most satisfactory in a first presentation of the object and scope of the Calculus. No comparisons have been made between this method and those of limits or of infinitesimals. This larger view of the Calculus, and of mathematical reasoning and processes in general, cannot readily be given with good results in the brief time allotted the subject in the general college course.

The immediate object of the Differential Calculus is the measurement and comparison of rates of change when the change is not uniform. Whether a quantity is or is not changing uniformly, however, the rate at any instant is determined in essentially the same manner; viz. by ascertaining what its change would have been in a unit of time had its rate remained what it was at the instant in question. It is this change which the Calculus enables us to determine, however complicated the law of variation may be. This conception of the nature of the problem is simple, and seems to afford the best foundation for further and more comprehensive study; while for those who are not to make a special study of mathematics it secures a more intelligent and less mechanical grasp of the problems involved than other methods whose conceptions and logic are not easily mastered in undergraduate courses.

My thanks are due to Professor Worthen, my colleague, for valuable suggestions and assistance in the reading of proofs.

ARTHUR SHERBURNE HARDY.

HANOVER, N.H., June 2, 1890.

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