CONSTRUCTION OF MORTALITY TABLES FROM THE RECORDS OF INSURED LIVES, ACTUARIAL STUDIES, NO. 2

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649019250

Construction of Mortality Tables from the Records of Insured Lives, Actuarial studies, No. 2 by Percy C. H. Papps & Ray D. Murphy

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

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Trieste

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PUBLISHED BY
THE ACTUARIAL SOCIETY OF AMERICA
256 BROADWAY, NEW YORK
1922

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Press of
The New Era Printing Company
Lancaster, Pa.

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CHAPTER I. INTRODUCTION.

A mortality table starts with a group of persons at a specified age and shows the number of survivors at each subsequent age. There is also generally set down the number dying in each year of age. The radix of the table, or the number living at the youngest age shown, is arbitrarily selected. Then, from the values of p_x for each age, the values of l_x may be computed successively for the higher ages by the relation, $l_x p_x = l_{x+1}$.*

As a general rule the construction of a mortality table based on the records of insured lives is effected by ascertaining the value of q_x for each age. This value is obtained by dividing θ_x by E_x, where θ_x represents the deaths and E_x the exposed to risk of death for the year between ages x and $x + 1$ in the data under observation.†

In practice some persons will be found who are under observation for only a part of a given year of age either because they enter the experience after the beginning of the year or because they pass out of it for causes other than death before the end of the year. In such cases each person is counted in computing E_x as a fraction equal to the proportion of the year under observation. The degree of accuracy with which such fractions are computed varies according to the method used in tabulating the data. A person who dies between ages x and $x + 1$ must be included in E_x as exposed to risk for the full year and not a fraction, because q_x represents the proportion of l_x persons alive at age x who will not be alive at age $x + 1$.

* See Institute of Actuaries' Text Book, Part II, Chapter I.

† As will be seen in Chapter III, θ_x may be the number of deaths and E_x the number of lives exposed to risk of death, or they may represent respectively the number of policies terminated by death and the number exposed to risk of termination by death; or lastly they may be the amount of insurance terminated by death and the amount exposed to risk. It is important to note, however, that both numerator and denominator in the fraction θ_x/E_x must always relate to the same kind of data.

It should be remembered that theoretically the numerical value of the radix selected and the resulting size of the values of l_x and d_x are of no significance, but that the relative values of these functions are of vital importance. The use of a large radix is advisable, however, so that when the calculated values of l_x and d_x are adjusted to the nearest integer the necessary error introduced is insignificant. This will also have a bearing on the limiting age (ω) since this would be the lowest age for which the unadjusted value of l_x is less than .5, unless fractions are shown.

When an applicant is accepted for insurance after being examined by the company's physicians, he is a "select" life. Among a number of such lives after the lapse of a few years there will be some whose health has become impaired to a greater or less degree, while others will remain as healthy as when first examined for insurance. The survivors of a body of select lives are therefore called "mixed" lives.

It follows that the rate of mortality of insured persons of a given attained age, say x, will not be the same among persons just insured at age x as among persons insured n years ago at age $x - n$. It has been found by experience that persons just insured at age x are subject to a lower rate of mortality than those also aged x, but insured at age $x - 1$. These latter, in turn, usually show a lower rate of mortality than those aged x, but insured at age $x - 2$. In general, for limited values of n, which vary in different experiences and in different age sections of a single experience, it is found that $q_{[x-\overline{n-1}]+\overline{n-1}} < q_{[x-n]+n}$ where the portion of the suffix within the square brackets indicates the age at issue, and the other portion, the duration since entry, the total being the present age, *i.e.*, x.

As n increases, the extent of the difference will be found to decrease, so that if $q_{[x-\overline{n-1}]+\overline{n-1}} = q_{[x-n]+n} - \delta_n$ then as n increases, δ_n will approach the limit zero.

If t be the greatest value of n for which the relation $q_{[x-\overline{n-1}]+\overline{n-1}} < q_{[x-n]+n}$ holds, this fact is expressed by saying that the effects of selection last for t years. Accidental fluctuations in the data, on which the mortality rates are based, are alone sufficient to prevent any exact determination of the value of t. An approximate value is all that can be expected. In practice, t may have a small value, as, for instance, in the case of residents in the tropics, where values as low as 2 years or even 1 year may be found; or it may have such a comparatively large value as 10 years or more, the latter figure applying in the case of the British Offices' Experience (1863–1893) under whole life participating policies, especially at the younger ages. It may be questioned whether the effects of selection ever entirely disappear or whether they become so merged with other influences, such as changes in sanitary conditions and in the mortality of the general population, that they are lost.* As a practical matter, however, we are warranted in assuming that they cease after a certain period.

Now, if a body of select lives all of a given age be observed, and the rate of mortality resulting during the first, second, third, etc., years of insurance be set forth for each year, the result will be a select table of mortality for that particular age at entry. If similar tables be prepared for each age at date of selection, we obtain a set of "select mortality tables."

It would involve much labor, however, to base calculations on such a set of select tables. A trial is therefore made to ascertain the effective period of selection beyond which the rate of mortality appears to depend only upon the attained age and may consequently be formed into one "ultimate" table. This may be done by a direct comparison of the values found for $q_{[x-n]+n}$ for different values of n as indicated above. Such a comparison may be confusing, however, because of the large number of values to be observed and the fluctuations in them, and it will usually be more satisfactory to determine by observation approximately where the line of division lies and then apply a different final test.

In describing the construction of the American Men Tables, it was stated that "the crude death rates were deduced for each of the first five insurance years, for the sixth and succeeding years combined, and for the eleventh and succeeding years combined. The expected deaths for each of the first ten insurance years were then calculated by graded rates of mortality based upon the data for the sixth and succeeding insurance years in order that the number of years for which medical selection lasted could be determined. It was seen that the material for the sixth and succeeding insurance years could be safely combined according to attained age."

A different method was used in compiling the $O^{[M]}$ table. The expectations of life were employed, as they would not be subject

* See *T. A. S. A.*, Vol. XIII, page 211, for a discussion of the effect on select tables of a variation in mortality during the period of investigation.

to fluctuations to the same degree as would the mortality rates for individual ages. On page 146 of "Account of Principles and Methods" of that experience are shown, for quinquennial groups of ages, the values of the expectations $e_{[x]}$, $e_{[x-5]+5}$, $e_{[x-10]+10}$, etc., to $e_{[x-25]+25}$. There are also given the values of $e_x^{[5]}$ and $e_x^{[10]}$, the expectations of life found by combining the data for the same attained age, but excluding the data for the first five and first ten years of duration respectively. If selection were still effective in the $(n+1)$th year, $e_{[x-n]+n}$ would be greater than $e_x^{[n]}$. It was decided for practical advantages to consider that the effect of selection had disappeared in ten years, although this did not appear to be true for all ages at entry.

A select and ultimate table may be set forth conveniently as shown by the following section of the $O^{[NM]}$ table.

Age at Entry.	Years Elapsed Since Date of Insurance.						Age Attained.
	0	1	2	3	4	5 or more.	
x.	$l_{[x]}$.	$l_{[x]+1}$.	$l_{[x]+2}$.	$l_{[x]+3}$.	$l_{[x]+4}$.	$l_{[x]+5}$.	$x+5$.
20	100,000	99,580	99,008	98,383	97,616	96,879	25
21	99,264	98,844	98,267	97,596	96,877	96,137	26
22	98,530	98,109	97,530	96,857	96,135	95,392	27
23	97,794	97,369	96,790	96,115	95,389	94,641	28
24	97,055	96,630	96,048	95,370	94,639	93,886	29
25	96,316	95,887	95,302	94,619	93,884	93,124	30
26	95,567	95,135	94,547	93,862	93,122	....	31
27	94,818	94,382	93,791	93,100	....	....	32
28	94,059	93,618	93,023	....	....	....	33
29	93,300	92,854	....	....	....	....	34
30	92,529	....	....	....	....	....	35
31	....	....	....	....	....	....	36
32	....	....	....	....	....	....	37
33	....	....	....	....	....	....	38

When the rates of mortality are obtained, the first line of the table may be started with the desired radix and the values successively computed across the first line and then down the last column. The second and subsequent lines may be calculated by working back from the ultimate column by means of the equality $\log l_{[x]+n-1} = \log l_{[x]+n} - \log p_{[x]+n-1}$.

If the data entering into both the select and ultimate sections of the table be combined, or, in other words, if a mortality table be formed according to age only, irrespective of the year of insurance, the result will be an "aggregate table."

While it may be known that the effects of selection last for several years, it may be thought desirable for practical purposes to construct a table of mortality excluding the experience of, say, the first two years only, without constructing the select tables corresponding to those first two years. Such a table is known as a "truncated" table. Every ultimate table is in a sense a truncated table, but the name "ultimate" is usually applied only to a table which forms the continuation of the select section of a mortality table. In the select section the rate of mortality is shown for the age attained, but modified according to the length of time elapsed after initial selection. In an aggregate or an ultimate table the rate of mortality is shown for each age attained, without modification.

It is desirable to consider the effects of the duration of insurance on the data entering into an aggregate table. It will be understood that the following remarks will apply generally but in a modified degree to a truncated table. By first obtaining a clear idea of the nature of these different forms of mortality tables, the student will be in a better position to grasp the significance of the various methods of investigating and collecting the data.

For the sake of illustration, let it be first assumed that at all ages the effects of selection will last for ten years only. Then, if an aggregate table be formed from the experience of a company that has been in business for only ten years, the resulting table will be composed only of lives which have not reached the ultimate rates of mortality. The rates of mortality shown will obviously be much less than will be the case when the table is based upon the total experience of an old company which has been many years in business; for in the latter case, the higher ultimate rates of mortality of the old business will be included, raising the aggregate rates for any given age above the lower mortality of the newer business for the same age. Again, if there be two companies of the same age, the aggregate tables formed from the experience of the respective companies will differ considerably if one company has recently been writing a much larger business in relation to its size than the other. The company having the larger proportion of select business in its aggregate data will, other conditions being the same, show the lower mortality experience. In making this statement it is assumed that the age distribution is similar and that the companies are subject to