

**ELEMENTS OF THE
DIFFERENTIAL CALCULUS,
WITH EXAMPLES AND
APPLICATIONS: A TEXT BOOK**

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Elements of the Differential Calculus, with Examples and Applications: A Text Book by W. E. Byerly

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W. E. BYERLY

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Alexander Givens 7

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DIFFERENTIAL CALCULUS,

WITH

EXAMPLES AND APPLICATIONS.

A TEXT BOOK

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PREFACE.

THE following book, which embodies the results of my own experience in teaching the Calculus at Cornell and Harvard Universities, is intended for a text-book, and not for an exhaustive treatise.

Its peculiarities are the rigorous use of the Doctrine of Limits as a foundation of the subject, and as preliminary to the adoption of the more direct and practically convenient infinitesimal notation and nomenclature; the early introduction of a few simple formulas and methods for integrating; a rather elaborate treatment of the use of infinitesimals in pure geometry; and the attempt to excite and keep up the interest of the student by bringing in throughout the whole book, and not merely at the end, numerous applications to practical problems in geometry and mechanics.

I am greatly indebted to Prof. J. M. Peirce, from whose lectures I have derived many suggestions, and to the works of Benjamin Peirce, Todhunter, Duhamel, and Bertrand, upon which I have drawn freely.

W. E. BYERLY.

CAMBRIDGE, October, 1879.

TABLE OF CONTENTS.

CHAPTER I.

INTRODUCTION.

Article.	Page.
1. Definition of <i>variable</i> and <i>constant</i>	1
2. Definition of <i>function</i> and <i>independent variable</i>	1
3. Symbols by which functional dependence is expressed	2
4. Definition of <i>increment</i> . Notation for an increment. An increment may be positive or negative	2
5. Definition of the <i>limit</i> of a variable	3
6. Examples of <i>limits</i> in Algebra	3
7. Examples of <i>limits</i> in Geometry	4
8. The fundamental proposition in the <i>Theory of Limits</i>	5
9. Application to the proof of the theorem that the area of a circle is one-half the product of the circumference by the radius	5
10. Importance of the clear conception of a <i>limit</i>	6
11. The velocity of a moving body. <i>Mean velocity</i> ; <i>actual velocity</i> at any instant; <i>uniform velocity</i> ; <i>variable velocity</i>	6
12. Actual velocity easily indicated by aid of the <i>increment</i> notation	7
13. Velocity of a falling body	7
14. The direction of the tangent at any point of a given curve. Definition of <i>tangent</i> as limiting case of <i>secant</i>	8
15. The <i>inclination</i> of a curve to the axis of X easily indicated by the aid of the <i>increment</i> notation	8
16. The inclination of a parabola to the axis of X	9
17. Fundamental object of the Differential Calculus	10

CHAPTER II.

DIFFERENTIATION OF ALGEBRAIC FUNCTIONS.

18. Definition of <i>derivative</i> . Derivative of a <i>constant</i>	11
19. General method of finding the derivative of any given function. General formula for a derivative. Examples	11

Article.	Page.
20. Classification of functions	12
21. Differentiation of the product of a constant and the variable; of a power of the variable, where the exponent is a positive integer	13
22. Derivative of a sum of functions	14
23. Derivative of a product of functions	15
24. Derivative of a quotient of functions. Examples	17
25. Derivative of a function of a function of the variable	18
26. Derivative of a power of the variable where the exponent is negative or fractional. Complete set of formulas for the differentiation of Algebraic functions. Examples	19

CHAPTER III.

APPLICATIONS.

Tangents and Normals.

27. Direction of tangent and normal to a plane curve	22
28. Equations of tangent and normal. Subtangent. Subnormal. Length of tangent. Length of normal. Examples	23
29. Derivative may sometimes be found by solving an equation. Examples	25

Indeterminate Forms.

30. Definition of infinite and infinitely great	26
31. Value of a function corresponding to an infinite value of the variable	26
32. Infinite value of a function corresponding to a particular value of the variable	27
33. The expressions $\frac{0}{0}$, $\frac{\infty}{\infty}$, and $0 \times \infty$, called indeterminate forms. When definite values can be attached to them	28
34. Treatment of the form $\frac{0}{0}$. Examples	28
35. Reduction of the forms $\frac{\infty}{\infty}$ and $0 \times \infty$ to the form $\frac{0}{0}$	30

Maxima and Minima of a Continuous Function.

36. Continuous change. Continuous function	31
37. If a function increases with the increase of the variable, its derivative is positive; if it decreases, negative	31
38. Value of derivative shows rate of increase of function	32
39. Definition of maximum and minimum values of a function	32

TABLE OF CONTENTS.

vii

Articles.	Page.
40. <i>Derivative zero at a maximum or a minimum</i>	33
41. Geometrical illustration	33
42. Sign of derivative near a zero value shown by the value of its own derivative	34
43. Derivatives of different orders	34
44. Numerical example	34
45. Investigation of a minimum	35
46. Case where the third derivative must be used. Examples	35
47. General rule for discovering maxima and minima. Examples	36
48. Use of auxiliary variables. Examples	38
49. Examples	39

Integration.

50. Statement of the problem of finding the <i>distance</i> traversed by a falling body, given the <i>velocity</i>	41
51. Statement of the problem of finding the <i>area</i> bounded by a given curve	41
52. Statement of the problem of finding the length of an arc of a given curve	42
53. <i>Integration. Integral</i>	44
54. <i>Arbitrary constant</i> in integration	44
55. Some formulas for direct integration	44
56. Solution of problem stated in Article 50	45
57. Example under problem stated in Article 51. Examples	46
58. Examples under problem stated in Article 52	48

CHAPTER IV.

TRANSCENDENTAL FUNCTIONS.

59. Differentiation of $\log x$ requires the investigation of the limit of $\left(1 + \frac{1}{n}\right)^n$	49
60. Expansion of $\left(1 + \frac{1}{n}\right)^n$ by the Binomial Theorem	50
61. Proof that the limit in question is the sum of a well-known series	50
62. This series is taken as the base of the natural system of logarithms. Computation of its numerical value	52
63. Extension of the proof given above to the cases where n is not a positive integer	53
64. Differentiation of $\log x$ completed	54
65. Differentiation of a^x . Examples	55

Article.	<i>Trigonometric Functions.</i>	Page.
66.	<i>Circular</i> measure of an angle. Reduction from <i>degree</i> to <i>circular</i> measure. Value of the <i>unit</i> in circular measure . . .	57
67.	Differentiation of $\sin x$ requires the investigation of the limit $\frac{\sin \Delta x}{\Delta x}$ and $\frac{1 - \cos \Delta x}{\Delta x}$	57
68.	Investigation of these limits	58
69.	Differentiation of the Trigonometric Functions. Examples .	59
70.	<i>Anti-</i> or <i>inverse</i> Trigonometric Functions	60
71.	Differentiation of the <i>Anti-Trigonometric</i> Functions. Examples	60
72.	<i>Anti-</i> or <i>inverse</i> notation. Differentiation of <i>anti-</i> functions in general	61
73.	The derivative of y with respect to x , and the derivative of x with respect to y , are reciprocals. Examples	62

CHAPTER V.

INTEGRATION.

74.	Formulas for <i>direct</i> integration	65
75.	Integration by <i>substitution</i> . Examples	66
76.	If $\int x$ can be integrated, $\int (a + bx)$ can always be integrated. Examples	67
77.	$\int_x \frac{1}{\sqrt{(a^2 - x^2)}}$. Examples	67
78.	$\int_x \frac{1}{\sqrt{(a^2 + x^2)}}$. Example	68
79.	<i>Integration by parts</i> . Examples	69
80.	$\int_x \sin^2 x$. Examples	69
81.	Use of <i>integration by substitution</i> and <i>integration by parts</i> in combination. Examples	70
82.	Simplification by an <i>algebraic transformation</i> . Examples . . .	71

Applications.

83.	Area of a segment of a circle; of an ellipse; of an hyperbola .	72
84.	Length of an arc of a circle	74
85.	Length of an arc of a parabola. Example	75

CHAPTER VI.

CURVATURE.

86.	<i>Total</i> curvature; <i>mean</i> curvature; <i>actual</i> curvature. Formula for actual curvature	77
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TABLE OF CONTENTS.

ix

Article.	Page.
87. To find <i>actual curvature</i> conveniently, an indirect method of differentiation must be used	77
88. The derivative of z with respect to y is the quotient of the derivative of z with respect to x by the derivative of y with respect to x	78
89. Reduced formula for <i>curvature</i> . Examples	78
90. <i>Osculating circle</i> . <i>Radius of curvature</i> . <i>Centre of curvature</i>	81
91. Definition of <i>evolute</i> . Formulas for <i>evolute</i>	82
92. <i>Evolute</i> of a parabola	83
93. Reduced formulas for <i>evolute</i> . Example	85
94. <i>Evolute</i> of an ellipse. Example	85
95. Every normal to a curve is tangent to the <i>evolute</i>	87
96. Length of an arc of <i>evolute</i>	88
97. Derivation of the name <i>evolute</i> . <i>Involute</i>	88

CHAPTER VII.

THE CYCLOID.

98. Definition of the cycloid	90
99. Equations of cycloid referred to the base and a tangent at the lowest point as axes. Examples	90
100. Equations of the cycloid referred to vertex as origin. Examples	92
101. Statement of properties of cycloid to be investigated	93
102. Direction of tangent and normal. Examples	93
103. Equations of tangent and normal. Example	94
104. Subtangent. Subnormal. Tangent. Normal	94
105. Curvature. Examples	95
106. <i>Evolute</i> of cycloid	96
107. Length of an arc of cycloid	97
108. Area of cycloid. Examples	98
109. Definition and equations of <i>epicycloid</i> and <i>hypocycloid</i> . Examples	99

CHAPTER VIII.

PROBLEMS IN MECHANICS.

110. Formula for <i>velocity</i> in terms of distance and time	102
111. <i>Acceleration</i> . Example. Differential equations of motion	102
112. Two principles of mechanics taken for granted	103
113. Problem of a body falling freely near the earth's surface	103