A NEW METHOD OF CALCULATING THE CUBIC CONTENTS OF EXCAVATIONS AND EMBANKMENTS

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A NEW METHOD

OF

CALCULATING THE CUBIC CONTENTS

OF

EXCAVATIONS AND EMBANKMENTS,

BY THE AID OF DIAGRAMS.

TOGETHER WITH

DIRECTIONS FOR ESTIMATING THE COST OF EARTHWORK.

BY

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THIRD EDITION.
Completely Revised and Enlarged.

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PREFACE.

ALTHOUGH the usual methods of obtaining correctly the cubic contents of Excavations and Embankments cannot be said to involve any *difficulty*, still they are certainly open to the objection of being very *tedious*.

Consequently, any device for diminishing the *labor*, without affecting the *accuracy*, of the operation, may justly be regarded as a desideratum of some importance; and, we believe, that the method by diagrams here proposed, will conduce to that result, both in the field and in the office.

It originated with the writer many years since; and was first published in 1851.

Should an objection be made to the admission of the transverse ground-slopes, as an element in the calculations, we can only reply that, practically, it is *at least* as accurate as that based upon the usual assumption, that the two outer heights or depths of a cross-section represent the exterior elevation of *straight lines*, drawn from those points to the centre-stake.

Sensible of the necessity of perfect accuracy in the Tables, they have been prepared with the greatest care; and have undergone so thorough a revision as to leave scarcely a doubt of their *entire reliability*.

A NEW METHOD

OF

CALCULATING THE CUBIC CONTENTS

OF

EXCAVATIONS AND EMBANKMENTS.

THERE is but one correct principle upon which to calculate the cubic contents of excavations and embankments; and that is, the *Prismoidal Formula*, or Rule; which is as follows:

Add together the areas of the two parallel ends of the prismoid, and four times the area of a section half-way between and parallel to them; and multiply the sum by one-sixth of the length of the prismoid, measured perpendicularly to its two parallel ends.

Since, in railroad measurements, the prismoids are generally 100 feet long, it becomes easier in practice to multiply the sum of the areas in square feet, by 100, (by merely adding two ciphers,) and to divide the product by 6; which amounts to the same thing as multiplying their sum by $\frac{1}{6}$th of 100 feet.

The very extended application of the prismoidal formula to other solids than such as are commonly understood by the term "prismoids," was first shown by Mr. Ellwood Morris, Civil Engineer, in a paper published in the Journal of the Franklin Institute, in 1840.

It embraces all parallelopipeds, pyramids, prisms, cylinders, cones, wedges, &c., whether regular or irregular, right or oblique; together with their frustums, when cut by planes parallel to their bases; in a word, *any solid whatever, which has two parallel ends, connected together by either plane, or by longitudinally unwarped surfaces.* It also applies to spheres, hemispheres, spheroids, paraboloids, &c.

In the cylinder and cone, the sides may be considered as consisting of an infinite number of infinitely narrow planes, unwarped longitudinally. In railroad cuttings, it rarely happens that the surface planes lying between two consecutive cross sections, 100 feet apart, are absolutely unwarped; yet, for practical purposes, they may very frequently be assumed to be so. When much warped, the cross sections must be taken closer together than 100 feet. *Upon a strict attention to this precaution depends the accuracy of earthwork measurements; the entire principle of which is embraced in the foregoing remarks.* No practicable method is perfectly accurate. All we can do in actual practice is, to take our stations at distances so near together that the intermediate solid shall be *very nearly* a prismoid, and then calculate it *as if a true* prismoid.

There are generally two circumstances under which it is necessary to compute the cubic contents on a public work; viz.: first, after a preliminary survey of one, or more *trial lines;* for the purpose of determining *approximately* their actual, or comparative costs; and, second, after the final adoption, and staking out the determined route, in order to know *precisely* the amount of work to be done.

The measurements for the latter are performed with more care, and attention to detail, than those of the former, inasmuch as upon them depend the payments to be made to the person who executes the work. They, moreover, involve considerations which cannot be attended to during a preliminary survey, without incurring an expenditure of time and labor, more than commensurate with the importance of the result.

When the ground is *level* transversely of the line of survey, there is no difficulty whatever in ascertaining the contents from a table of *level-cuttings*, previously calculated; but when it is *inclined* or *irregular* transversely, the calculations have hitherto been attended with considerable labor.

The following method by diagrams will we trust, be found to render the operations in the last cases, if not as simple and expeditious as in those of level ground, at least much more so than the usual ones. It dispenses with a great deal of calculation; and is, therefore, comparatively free from errors arising from that source.

METHOD OF USING THE DIAGRAMS.

The construction of the diagrams is extremely simple, notwithstanding that, at first sight, they appear somewhat complex. They are but few in number, since any particular road will generally require but three or four, which may be prepared by one person in a few days. Before proceeding to explain the manner of drawing them, we will give one or two examples of their use, that the reader may see the object aimed at, and to what extent it is attained.

Example 1. Suppose that in a roadway of 28 feet wide, and with side-slopes of 1½ to 1, the cutting at a certain station is 20 feet; and that the ground, instead of being level transversely, inclines at an angle of 15°

Turn to the diagram, Plate IX., for a roadway 28 feet wide, with side-slopes of 1½ to 1: place a finger on the centre line, at the height of 20 feet, and run it along up the curved line which commences at that point, until it strikes the inclined line marked 15°. It will be seen at once that the two coincide at the height of 22·8 feet: *and this is the depth of the equivalent level cutting, which would have precisely the same area as the section under consideration.*

All such cases may therefore be instantly, and without any calculation whatever, reduced to others of *equivalent level cuttings.*

This constitutes the main feature of the principle involved in the diagrams.

Had the depth been 20·3, or other decimal of a foot, the proceeding would have been the same as with the 20 feet; and the equivalent level cutting would be found on the inclined line 15°, at the distance of ·3 of a foot (estimated by eye) above the curved line 20.

Example 2. Using the same diagram; let the depth of cutting be 2 feet, and the transverse slope of the ground 20°. Here, placing a finger on the centre line, at the height of 2 feet, and running it along the curved line commencing at that point, it will be found that before reaching the inclined line of 20°, it encounters the *dotted* curved line drawn near the bottom of the diagram. When this occurs, we know that the ground-slope cuts the roadway, forming a cross section, partly in excavation, and partly in embankment, as in fig. 9.

This is a most useful check; for in such cases, the contents cannot be obtained by means of the diagram; but recourse must be had to a figure of the section drawn for the purpose; as must also be the case when the ground is *irregular* transversely. A simple method of proceeding, in all such cases, will be given further on.

On the page opposite each diagram, is a table of cubic yards for level cuttings, and for lengths of 100 feet. By means of these tables, the cubic contents may at once be taken out, when the equivalent level cuttings at both ends of a station are equal, and the ground-slope between them uniform: but if the equivalent level cuttings at the two ends of the station are unequal, then the prismoidal rule must be applied; thus,

Suppose the equivalent level cutting at one end to be 20 feet, and at the other 25 feet, and the intervening ground-slope uniform. Then the equivalent level cutting at a point half-way between them would be 22½ feet. Therefore, the cubic content will be equal to one-sixth of the sum of those corresponding to each of the two end depths, and of four times that of the centre depth; that is,

Cubic content by table 9, for 20 feet depth,	=	4296	cubic yards,	
" " " " 25 "	=	6065	"	"
Four times " " $22\frac{1}{2}$ or 4 times 5146 } "	=	20584	"	"
		6)30945		
Cubic yards contained in the station,	=	5157·5		

These tables are carried to depths or heights of 60 feet; but in the subsequent table No. 15, they are extended to 170 feet. As these extended quantities will be but seldom referred to, they are calculated only to whole feet; but the amount corresponding to any fraction of a foot may be found with sufficient accuracy for practice, by simple proportion.

It will be perceived that, instead of the *areas* corresponding to the different depths of cutting, or heights of filling, our tables give the *cubic yards* corresponding to those areas, for lengths of 100 feet. For the purposes of calculating cubic contents, these solidities may evidently be used instead of the areas; but for such cases as require the areas themselves, a table (No. 17) of such is added. Its use will be shown further on.

For rough preliminary estimates of trial lines, the labor may be much reduced by taking from the tables, the cubic content corresponding to the *average* of the equivalent level cuttings at the two ends. This mode is not mathematically correct, and should never be resorted to for final estimates; but it will be sufficiently approximate (always a little deficient) for such cases as occur in ordinary cuttings and fillings; and even *where the depths at the two ends do not differ more than about* 5 *feet; nor the ground-slopes differ more than about* 5°; said slopes being in the same direction.

For instance, in the foregoing example, the correct contents of the station 20 feet deep at one end, and 25 feet at the other, were found to be 5157·5 cubic yards; while, by this approximating mode, the contents of an *average* level depth of $22\frac{1}{2}$ feet, would be 5146 cubic yards; or but $11\frac{1}{2}$ yards less than the truth.

Or, for true prismoids, or even within the foregoing limits of no greater differences than 5 feet in depth; and 5° in slope at the two ends of a 100 feet station, the slopes being in the same direction, we may add together the tabular contents corresponding to the two equivalent level depths at the ends of the station, and divide their sum by 2. The content thus found will not be as approximate, however, as that by the first method; but will be *too great* by precisely *twice* the quantity that the other is *too small*. Thus, in the foregoing example, we should have for a true prismoid,

Depth.	Cubic yards.
20	4206
25	6065
	2)10361
	5180 cubic yards = approx. content,

or 23 yards in excess of the true content, $5157\frac{1}{2}$ yards; or twice the deficiency ($11\frac{1}{2}$ yards) of the preceding method.

These examples merely show that in railroad work, and within limits of frequent occurrence, we may calculate the content of a true prismoid by either of these approximate modes, with sufficient accuracy for rough preliminary, or comparative estimates. We have in neither instance given the actual content of a solid whose transverse slopes differ at its two ends. Said content would be farther from the truth than in our examples; where, by the first method, the error is but 1 yard in about 450; and in the second, 1 in about 225; whereas the average of a number of stations in which the slopes at the two ends differ on an average $2\frac{1}{2}°$, and in no case more than 5°, would probably be in error by about 1 yard in 100 too little, by the first method; and 1 yard in 50 too much, by the second.

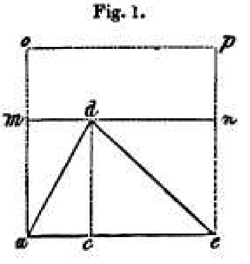
For *final* estimates, however, we should make our stations so short that the ground surface of the included solid may be considered unwarped longitudinally, and then use the prismoidal rule.

PRINCIPLE ON WHICH THE METHOD IS BASED,

To find the sides of a triangle of which only the area and the angles are given.

Rule.—In any plane triangle, as the product of the sines of any two of the angles, is to the product of radius by the sine of the remaining angle, so is twice the area of the triangle, to the square of the side lying between the two angles first taken.

Fig. 1.



Demonstration.—Let *a d e* be a triangle, in which we have given, its area, and its three angles: it is required to find any side, as *a e*.

By trigonometry we have the two following proportions:—

Sine of *a* (*d c*) : Radius (*a d*) :: *d c* : *a d*; also

Sine of *e* the angle opp. *a d* : Sine of *d* the angle opp. *a e* :: *a d* : *a e*.

By multiplication of these two proportions, we have—

Sine of $a \times$ Sine of e : Rad. $\times$ Sine of d :: $d c \times a d$: $a e \times a d$; or,