

**ELEMENTS OF THE
DIFFERENTIAL AND INTEGRAL
CALCULUS; WITH EXAMPLES
AND APPLICATIONS**

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Elements of the Differential and Integral Calculus; With Examples and Applications by James M. Taylor

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JAMES M. TAYLOR

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PREFACE.

THE object of the following treatise is to present simply and concisely the fundamental problems of the Calculus, their solution, and more common applications.

Since variables are its characteristic quantities, the first fundamental problem of the Calculus is, *To find the ratio of the rates of change of related variables.* To enable the learner most clearly to comprehend this problem, the author has employed the conception of rates, which affords finite differentials and the simplest demonstration of many principles. The problem of Differentiation having been clearly presented, a general method of its solution is obtained by the use of limits. This order of development avoids the use of the indeterminate form $\frac{0}{0}$, and secures all the advantages of the differential notation. Many principles are proved, both by the method of rates and that of limits, and thus each is made to throw light upon the other.

In a final chapter, the method of infinitesimals is briefly presented; its underlying principles having been previously established.

The chapter on Differentiation is followed by one on Integration; and in each, as throughout the work, there

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are numerous practical problems in Geometry and Mechanics, which serve to exhibit the power and use of the science, and to excite and keep alive the interest of the student.

In writing this treatise, the works of the best American, English, and French authors have been consulted; and from these sources the most of the examples and problems have been obtained.

The author is indebted to Professors J. E. OLIVER and J. MCMAHON of Cornell University, and Professor O. ROOT, Jr., of Hamilton College, for valuable suggestions; and to Messrs. J. S. CUSHING & Co. for the typographical excellence of the book.

J. M. TAYLOR.

HAMILTON, N. Y.,
Nov., 1884.

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