ELEMENTS OF PLANE TRIGONOMETRY, WITH ITS APPLICATION TO MENSURATION OF HEIGHTS AND DISTANCES, SURVEYING AND NAVIGATION

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WILLIAM SMYTH

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SURVEYING AND NAVIGATION.

BY

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ADVERTISEMENT.

In the following treatise the object has been to present the subject of Plane Trigonometry in a natural and connected order, and in general to lead the learner to feel the want of a new principle before proceeding to its investigation. After the necessary definitions the learner is taught to solve a triangle by construction, the method which naturally first occurs. From the obvious imperfection of this method, he is led to see the necessity, where accuracy is desired, of a solution by arithmetical calculation. But here a difficulty presents itself, arising from the impracticability of uniting in the same calculation the incongruous elements of straight lines and angles. In the effort to surmount this difficulty he is led to see the manner in which, first the sines and cosines, and secondly the tangents and secants are introduced, and made subservient to the object before him. These are found to be simply parts in an indefinite series of right angled triangles, themselves susceptible of calculation, and by means of which other similar triangles, and finally all plane triangles may be solved. The theory of the trigonometrical tables is thus developed, and the object proposed, the complete solution of a plane triangle, accomplished.

The principles of Plane Trigonometry having thus been obtained, are next applied to the mensuration of inaccessible Heights and Distances, to Surveying and Navigation, and in a manner adapted, it is hoped, to exhibit clearly their great practical value.

In the Surveying, in particular, great care has been taken in the preparation of examples and arrangement of the subject, gradually to lead the learner from the more simple survey of an ordinary field to that of routes for canals and railroads, the survey of the Public Lands, the method of exhibiting on paper the contour and accidents of the earth's surface, the preparing of maps of towns, counties, &c.; and finally the process by which the circumference of the earth is determined.

The work will form a part of the College course of Mathematics. Constant reference, however, has been had in its preparation to the wants of pupils in our Academies and High Schools, for whose use it will be found, it is believed, sufficiently simple.

WM. SMYTH.

Bowd. Coll., Nov. 1852.

CONTENTS.

BOOK I. PLANE TRIGONOMETRY.

BOOK II. SURVEYING.

BOOK III. NAVIGATION.

PLANE TRIGONOMETRY.

SECTION I.

PRELIMINARY REMARKS.

1. A figure bounded by three lines is called a *triangle*.

2. A triangle ABC (fig. 1), in which the bounding lines are straight lines, is called a *plane triangle*.

3. A plane triangle DEF (fig. 2), in which one of the sides DE forms with another DF a right angle, is called a *right angled triangle*. Any other triangle ABC (fig. 1), in which neither of the angles is a right angle, is called an *oblique angled triangle*.

4. In the right angled triangle DEF (fig. 2), the part DF opposite the right angle, is called the *hypothenuse;* the other two parts DE, EF are called the *sides;* sometimes also the *perpendicular and base*.

5. In a plane triangle there are six parts, viz. three angles and three sides. In order to determine the others, certain of these parts must be given.

6. A single part, it is evident, is not sufficient to determine the rest. Let us begin with two.

And first let the lines a and b (fig. 3), be two of the sides of a triangle. The lines a and b, it is evident, may make with each other any angle whatever. They may, therefore, be sides in an indefinite number of triangles, ABC, ABC,′ &c. The other parts of a triangle cannot, therefore, be determined, when two of the sides are the only parts which are given.

Again, let a side AB, and an angle A (fig. 4), be two given parts of a triangle. The other parts cannot be determined by these; for the side AB and the angle A may belong to an indefinite number of triangles ABC, ABC,′ &c.

Moreover, if two of the angles, or, which is the same thing, if all the angles are given, the remaining parts will still be undetermined; for in the triangle ABC (fig. 5) if the lines B′C′, B″C″, &c. are drawn parallel to BC, an indefinite number of triangles ABC, AB′C′, &c. may be formed, in each of which the angles will be respectively equal, while the sides are of different magnitudes.

In order then to determine the remaining parts of a triangle, *three, at least, of the parts must be given, one of which must be a side.*

7. It will be seen in what follows that the remaining parts of a triangle may always be determined, when three of the parts are given, provided that one of these is a side.

That branch of science which teaches how to find the remaining parts of a plane triangle from certain parts which are given, is called *Plane Trigonometry.*

SECTION II.

SOLUTION OF TRIANGLES BY GEOMETRICAL CONSTRUCTION.

8. The method of solving a triangle from certain of the parts which are given, which first suggests itself, is, to con-

struct the triangle by means of the given parts, and then to measure the required parts, in the use of the instruments by which the triangle is constructed.

9. Among the parts of a triangle there are two kinds of quantities, viz. sides and angles, for each of which measures must be found.

10. The sides of a plane triangle being right lines, are expressed in the usual linear measures, as feet, yards, &c. To represent these measures for the purpose of calculating a triangle geometrically, any line AB (fig. 6) may be divided into a convenient number of equal parts; one of these being considered as unity may be further divided into equal portions for fractional parts of unity. A line divided in this manner is called a *scale of equal parts.*

Thus in figure 6, the line AB is divided into eleven equal portions; of these the extreme one on the left is divided into ten equal parts, the remainder being numbered 1, 2, 3, &c. The principal divisions of this scale may be considered as feet, miles, &c.; then the subdivisions will be tenths of a foot, mile, &c. The principal divisions may also be regarded as ten feet, ten miles, &c.; in this case the subdivisions will represent feet, miles, &c.

A convenient construction for a scale of equal parts is represented figure 7. It consists, when intended for a decimal scale, of eleven lines drawn parallel to each other at equal distances, and divided into convenient portions by perpendicular lines. In the extreme divison on the left, the upper and lower lines are divided into ten equal parts, and the subdividing points are connected by *diagonal* lines, that is, by lines drawn from the first point on the lower line to the second on the upper. Then by similar triangles $ac : ab : : cd : bo$, that is, cd is one tenth of the subdivisions, or one hundreth of the primary divisions of the scale. In like manner, the corresponding portion on the second parallel line may be shown to be two hundreths, and that on the third parallel line three hundreths of the primary divisions of the scale.

Let it be required, for example, to take off the number 255 from this scale. Considering the subdivisions as containing ten each, we extend the dividers from figure 2 of the principal divisions to 5 of the subdivisions for 250; then by opening the dividers to the corresponding extent on the fifth of the parallel lines, we shall obtain the length required.

11. In order to estimate the magnitude of the angles, it would seem more natural to refer them to the right angle, as the unit of measure; since, in this case, the quantities to be measured would be the same in kind with the quantity assumed as their measure. It is found, however, more convenient in practice to measure angles by arcs of circles. In the circle AC (fig. 8), in whatever ratio the angle DBC at the centre increases or diminishes, the arc CD, on which it stands, increases or diminishes in the same ratio (Geom. B. 3); the arc CD may therefore be assumed, as the measure of this angle. In like manner any other concentric arc EF, intercepted by the sides of the angle DBC, may be used as its measure. The arc CD, indeed, considered as a magnitude of a certain length, is manifestly greater than the arc EF. In the measure of angles, however, it is not the absolute value of the arcs, which we regard, but merely their ratio to an entire circumference. Since then the arc EF is to the circumference EG, as the arc CD to the circumference CA; considered as parts of the entire circumferences, to which they respectively belong, the arcs CD and EF are equal in quanity. *The measure of an angle is then the arc of a circle, having its centre in the vertex of the angle, and intercepted by the lines, which form its sides.* In the measure