

**ELEMENTS OF PLANE AND  
SPHERICAL TRIGONOMETRY:  
WITH THEIR APPLICATIONS TO  
MENSURATION, SURVEYING,  
AND NAVIGATION**

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Elements of Plane and Spherical Trigonometry: With Their Applications to Mensuration, Surveying, and Navigation by Elias Loomis

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**ELIAS LOOMIS**

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ELEMENTS  
OF  
PLANE AND SPHERICAL  
TRIGONOMETRY,

WITH THEIR APPLICATIONS TO  
MENSURATION, SURVEYING, AND  
NAVIGATION.

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A "COURSE OF MATHEMATICS."

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## P R E F A C E.

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THE following treatise constitutes the third volume of a course of Mathematics designed for colleges and high schools, and is prepared upon substantially the same model as the works on Algebra and Geometry. It does not profess to embody every thing which is known on the subject of Trigonometry, but it contains those principles which are most important on account of their applications, or their connection with other parts of a course of mathematical study. The aim has been to render every principle intelligible, not by the repetition of superfluous words, but by the use of precise and appropriate language. Whenever it could conveniently be done, the most important principles have been reduced to the form of theorems or rules, which are distinguished by the use of italic letters, and are designed to be committed to memory. The most important instruments used in Surveying are fully described, and are illustrated by drawings.

The computations are all made by the aid of natural numbers, or with logarithms to six places; and by means of the accompanying tables, such computations can be performed with great facility and precision. This volume, having been used by several successive classes, has been subjected to the severest scrutiny, and the present edition embodies all the alterations which have been suggested by experience in the recitation room.





# CONTENTS.

## BOOK I.

### THE NATURE AND PROPERTIES OF LOGARITHMS.

Nature of Logarithms .....	7
Description of the Table of Logarithms.....	9
Multiplication by Logarithms.....	14
Division by Logarithms.....	14
Involution by Logarithms.....	17
Evolution by Logarithms.....	17
Proportion by Logarithms.....	18

## BOOK II.

### PLANE TRIGONOMETRY.

Sines, Tangents, Secants, &c., defined.....	20
Explanation of the Trigonometrical Tables.....	23
To find Sines and Tangents of small Arcs.....	29
Solutions of Right-angled Triangles.....	32
Solutions of Oblique-angled Triangles.....	36
Instruments used in Drawing.....	42
Geometrical Construction of Triangles.....	46
Values of the Sines, Cosines, &c., of certain Angles.....	48
Trigonometrical Formulae.....	52
Computation of a Table of Sines, Cosines, &c.....	57

## BOOK III.

### MENSURATION OF SURFACES AND SOLIDS.

Areas of Figures bounded by Right Lines.....	59
Area of a Regular Polygon.....	64
Quadrature of the Circle and its Parts.....	66
Mensuration of Solids.....	71
Rail-way Excavations or Embankments.....	77
Regular Polyhedrons.....	81
The three Round Bodies.....	84
Area of a Spherical Triangle.....	88

## BOOK IV.

### SURVEYING.

Definitions.....	94
Instruments for measuring Angles.....	91

	Page
Explanation of the Vernier .....	94
Description of the Theodolite .....	95
Heights and Distances .....	97
The Determination of Areas .....	103
Plotting a Survey .....	104
The Traverse Table .....	106
To find the Area of a Field .....	109
Trigonometrical Surveys .....	114
Variation of the Needle .....	117
Leveling .....	119
Topographical Maps .....	123
Setting out Rail-way Curves .....	127
Surveying Harbors .....	130
The Plane Table .....	132
To determine the Depth of Water .....	133

## BOOK V.

## NAVIGATION.

Definitions, &c. ....	135
Plane Sailing .....	138
Traverse Sailing .....	141
Parallel Sailing .....	144
Middle Latitude Sailing .....	146
Mercator's Sailing .....	149
Nautical Charts .....	153

## BOOK VI.

## SPHERICAL TRIGONOMETRY.

Right-angled Spherical Triangles .....	155
Napier's Rule of the Circular Parts .....	158
Examples of Right-angled Triangles .....	160
Oblique-angled Spherical Triangles .....	162
Examples of Oblique-angled Triangles .....	165
Trigonometrical Formulae .....	171
Sailing on an Arc of a Great Circle .....	174

# TRIGONOMETRY.

## BOOK I.

### THE NATURE AND PROPERTIES OF LOGARITHMS.

ARTICLE 1. Logarithms are numbers designed to diminish the labor of Multiplication and Division, by substituting in their stead Addition and Subtraction. All numbers are regarded as powers of some one number, which is called the *base* of the system; and the exponent of that power of the base which is equal to a given number, is called the logarithm of that number.

The base of the common system of logarithms (called, from their inventor, Briggs' logarithms) is the number 10. Hence all numbers are to be regarded as powers of 10. Thus, since

$10^0=1,$	0	is the logarithm of 1	in Briggs' system;
$10^1=10,$	1	“ “	10 “ “
$10^2=100,$	2	“ “	100 “ “
$10^3=1000,$	3	“ “	1000 “ “
$10^4=10000,$	4	“ “	10,000 “ “
&c.,		&c.,	&c.;

whence it appears that, in Briggs' system, the logarithm of every number between 1 and 10 is some number between 0 and 1, *i. e.*, is a proper fraction. The logarithm of every number between 10 and 100 is some number between 1 and 2, *i. e.*, is 1 plus a fraction. The logarithm of every number between 100 and 1000 is some number between 2 and 3, *i. e.*, is 2 plus a fraction, and so on.

(2.) The preceding principles may be extended to fractions by means of negative exponents. Thus, since

$10^{-1}=0.1,$	-1	is the logarithm of 0.1	in Briggs' system;
$10^{-2}=0.01,$	-2	“ “	0.01 “ “
$10^{-3}=0.001,$	-3	“ “	0.001 “ “
$10^{-4}=0.0001$	-4	“ “	0.0001 “ “
&c.,		&c.,	&c.