AN ELEMENTARY COURSE OF PRACTICAL MATHEMATICS. FOR THE USE OF SCHOOLS. PART III

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JAMES ELLIOT

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Trieste

AN ELEMENTARY COURSE

OF

PRACTICAL MATHEMATICS,

FOR THE USE OF SCHOOLS.

PART III.

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AUTHOR OF "A COMPLETE TREATISE ON PRACTICAL GEOMETRY AND MENSURATION,"
"A PRACTICAL TREATISE ON THE NATURE AND USE OF LOGARITHMS,
AND ON PLANE TRIGONOMETRY," ETC.

LOGARITHMS AND PLANE TRIGONOMETRY.

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PRACTICAL MATHEMATICS.

PART THIRD.

LOGARITHMS

AND

PLANE TRIGONOMETRY.

CONTENTS OF PART III.

LOGARITHMS

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CHAPTER I.

DEFINITIONS AND EXPLANATIONS OF LOGARITHMS.

If we take any series of numbers in arithmetical progression, commencing with 0, as 0, 1, 2, 3, &c., and another series in geometrical progression, commencing with 1, as 1, 2, 4, 8, &c.; and if we then place the two series together, thus—

0,	1,	2,	3,	4,	5,	6,	&c.
1,	2,	4,	8,	16,	32,	64,	&c.

then the numbers in the arithmetical series are said to be the *Logarithms* of the corresponding numbers in the geometrical series—the latter being called the *Natural Numbers.* Thus, taking the two preceding series, the logarithm of 64 is 6; the logarithm of 32 is 5; while the natural number corresponding to the logarithm 6 is 64; and so on.

In the specimen just exhibited we have taken two ascending progressions; but they might, equally well, have been two descending progressions, or the one descending and the other ascending. Logarithms, however, as now* used in practice, are limited to the case of two progressions, either both ascending or both descending;—the former giving the logarithms of integers,—the latter, of fractional numbers.

* It was not so in the table of logarithms first given to the world by their inventor.

In the same specimen, also, we have taken an arithmetical progression whose common difference is 1, and a geometrical progression having 2 for the common ratio of its terms; but progressions having any other common difference and ratio would have answered equally well. Thus, we may take the two following:—

0,	2,	4,	6,	8,	10,	&c.
1,	3,	9,	27,	81,	243,	&c.

Here, also, the numbers in the upper line are the logarithms of those in the lower. In the latter case, however, we have what is called a different *System* of Logarithms from that in the former. In the former system, 6 is the logarithm of 64; in the latter, 6 is the logarithm of 27.

There may thus be as many different systems as we please; but for practical use, it is necessary to select and adhere to one. That usually employed now, called *Briggs's System*, is the following:—

Logarithms...0,	1,	2,	3,	4,	&c.	
Numbers,.....1,	10,	100,	1000,	10000,	&c.	

in which a common difference of 1 in the arithmetical series corresponds to the common ratio 10 in the geometrical.

The common ratio in the geometrical series corresponding to the common difference of 1 in the arithmetical series, is called the *Basis* of the system. Thus, the basis of Briggs's system is 10; the basis in the specimen first exhibited is 2; and the basis in the second is $\sqrt{3}$.

In all the above instances it will be observed that the series of natural numbers is not complete. Thus, in the first example, we want the numbers 3, 5, 6, 7, 9, 10, &c., and their logarithms. In the last, we have no natural numbers between 1 and 10, between 10 and 100, &c., and consequently no logarithms corresponding to such numbers. To supply the logarithms of the intermediate numbers, so as to obtain a complete table, requires skilful applications of the highest principles of arithmetic, and immense labour. It is not the province of this work to explain the methods employed for that purpose; and the labour is now no longer necessary, since the calculation has been repeatedly made already, and the whole series of natural numbers, with their corresponding logarithms, have been arranged in tables ready for use. They have also been laid down on straight rules, called *Gunter's Scales* from their inventor;

on *Sliding Rules* of various kinds; and on *Circular Scales*, and *Circular Sliding Rules*.

It now remains to explain what the uses of logarithms are.

The operations of multiplication, division, and extraction of roots, by common arithmetic, become tedious—and combinations of these still more so—when the numbers are represented by many figures. The object of logarithms is to facilitate such operations. They were first employed systematically for that purpose by Baron Napier of Merchiston, who, after trying other ingenious devices, at last happily discovered this useful auxiliary to the arithmetician.

Logarithms have also other uses, which were not at first contemplated by their inventor; but these belong to the higher branches of Mathematics, and may be found explained in works on the Integral Calculus.

To show, in a simple manner, how tables of logarithms may be applied to the purposes of multiplication, division, &c., let us resume the two series with which our illustration commenced,—viz.,

Logarithms...	0,	1,	2,	3,	4,	5,	6,	&c.
Numbers......	1,	2,	4,	8,	16,	32,	64,	&c.

Let it now be required to multiply the number 16 by 4: we take their corresponding logarithms standing above them, namely 4 and 2, and *add* them together: their sum is 6, to which we look forward in the line of logarithms: under the 6 we find 64, which is the product we want. If the scholar choose to carry out the two series further, and to try a variety of other numbers, he may find their products in the same manner, without the trouble of multiplying. A little thought will suggest to him the reason; and that reason he will more readily perceive, if he resolve the numbers in the lower line into their factors, writing the two series thus—

0,	1,	2,	3,	4,	5,	6,	&c.
1,	2,	2.2,	2.2.2,	2.2.2.2,	2.2.2.2.2,	2.2.2.2.2.2,	&c.*

He will then see that each number in the upper line expresses the number of factors in the corresponding term of

* The point between the figures is used for the sign of multiplication, the same as ×. Thus 2.2.2 = 8.

the lower line. Now, the scholar must be already familiar with a principle commonly employed in multiplication,— that to multiply by any number is the same as to multiply by its several factors successively: thus, to multiply by 96 gives the same result as to multiply by 12 and again by 8. So, when we are required to multiply 16 by 4, it is the same as to multiply by 2 and by 2; thus $16 \times 4 = 16 \times 2.2$. But 16 is $= 2.2.2.2$. Therefore 16×4 is $= 2.2.2.2 \times 2.2$, or $= 2.2.2.2.2.2$. That is, we have the factor 2 as many times in the product as in the multiplier and multiplicand together; or the logarithm of the product will be equal to the sum of the logarithms of the multiplicand and multiplier.

Again: suppose it were required to divide 64 by 16; take their logarithms, 6 and 4, in the above series; the *difference* of these is two. We look for 2 in the line of logarithms, and under it we find 4, which is the quotient required. The student may, in like manner, try the process with a variety of other examples. The reason is similar to that of the process for multiplication.

It will be seen that, if we had our two series complete, we might find the products of any two numbers by adding their logarithms together; and their quotient by taking the difference of their logarithms.

Again: since the square of any number is the product of that number by itself, such square may be found by *doubling* the logarithm of the number. Thus, to square 8, we take its logarithm, which is 3 in the above system; twice 3 is 6; under 6 we find 64, which is the square required.

Consequently, the square root of any number may be found by *halving* the logarithm of the number. Thus, to extract the square root of 64, we take its logarithm, 6; the half of 6 is 3; under 3 we find 8, for the root.

To show these principles, we have selected one particular system of logarithms; but they might have been shown equally well in any other system.

The preceding are the *first principles* of logarithms: from these the reasons of the more intricate of the operations explained in the following problems are almost self-evident deductions. Such operations are: the multiplication of several factors; involution and evolution, where powers and roots higher than the second are sought; and processes combining multiplication, division, involution, and evolution.