

**ELEMENTS OF PLANE AND SPHERICAL  
TRIGONOMETRY, WITH ITS  
APPLICATIONS TO THE PRINCIPLES  
OF NAVIGATION AND NAUTICAL  
ASTRONOMY. WITH THE LOGARITHMIC  
AND TRIGONOMETRICAL TABLES**

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Elements of Plane and Spherical Trigonometry, with Its Applications to the Principles of Navigation and Nautical Astronomy. With the Logarithmic and Trigonometrical Tables by J. R. Young

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**J. R. YOUNG**

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NAVIGATION AND NAUTICAL ASTRONOMY;  
WITH THE  
LOGARITHMIC AND TRIGONOMETRICAL TABLES.

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BY <sup>the author</sup> J. R. YOUNG,

PROFESSOR OF MATHEMATICS IN BELFAST COLLEGE,

And Author of "Elements of Geometry," "Treatise on Algebra," "Elements of Analytical Geometry," "Mathematical Tables," "Computation of Logarithms," "Elements of the Differential Calculus," "Elements of the Integral Calculus," and "Elements of Mechanics."

TO WHICH ARE ADDED SOME ORIGINAL RESEARCHES IN

SPHERICAL GEOMETRY;

BY T. S. DAVIES, F.R.S. LOND. AND EDINB.

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1833.

## PREFACE.

It is the design of this treatise to establish the theory of Plane and Spherical Trigonometry analytically, and to present that theory, together with some of its most interesting and valuable applications, in a form fitted for elementary instruction.

Of late years several analytical works on Trigonometry have been published in this country; but, as they are confined almost entirely to the theory of the subject, it may be questioned whether, to many young students, they prove much else than so many collections of mere algebraical exercises. Yet a book upon so practical a subject as Trigonometry ought undoubtedly to be something more than this, and ought not to be considered as complete when the various calculations which the science involves, and which its name implies, are wholly omitted.

The symbolical expression of a practical rule, in algebraic language, will often, to the young student, but indistinctly point out the numerical operation. Those much occupied

in mathematical instruction know full well that a learner may readily yield his assent to every step of an algebraic process, be fully satisfied as to the truth of the result to which it leads, may even clearly see a valuable truth involved in it, and may yet be very far from perceiving how to turn it to account in any case of actual calculation. Indeed, algebraical formulas, transform them as we will, cannot always be made to indicate the best mode of arithmetical arrangement; and yet much, as regards facility of operation, depends upon this arrangement in many parts of practical mathematics, but especially in Trigonometry.

In the present volume, therefore, both the theory and the practice of the science have been introduced, every practical formula being illustrated by examples of the numerical calculation, arranged in the proper form. This plan of combining practice with theory, in works like the present, was always adopted by the earlier English writers, and it is to be regretted that recent authors have, in their admiration of foreign methods, departed so widely, in this respect, from the example of their predecessors, dwelling so much as they do upon the symbols, and so little upon the things signified.

In addition to the practical illustration of formulas, a distinct part of the work is devoted to the principles of Navigation and Nautical Astronomy, in which will be found a very short and convenient method of clearing the Lunar Distance, for the purpose of ascertaining the Longitude at

Sea. This method is probably new, although, as the analytical expression for it occurs during the investigation of the well known formula of *Borda*, it is equally probable that it has been noticed before.

The supplement appended to the treatise is from the pen of my valued and accomplished friend, *T. S. Davies, Esq.* Fellow of the Royal Society of Edinburgh, and of the Royal Astronomical Society of London. It will be found to contain several new and interesting researches, which cannot fail to prove acceptable both to the inquiring student and to the more advanced analyst.

J. R. YOUNG.

January 1, 1833.

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