ELEMENTARY TREATISE ON DETERMINANTS: WITH THEIR APPLICATION TO SIMULTANEOUS LINEAR EQUATIONS AND ALGEBRAICAL GEOMETRY

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CHARLES L. DODGSON

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ON

DETERMINANTS

WITH THEIR APPLICATION TO

SIMULTANEOUS LINEAR EQUATIONS
AND ALGEBRAICAL GEOMETRY.

BY

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PREFACE.

OF the seventy Propositions contained in the following treatise, ten are substantially taken from Baltzer's treatise on Determinants; also the Geometrical Tests, given in Chapter VIII, are to be found in most works on Algebraical Geometry: the rest of the matter is, so far as I know, original, and consists of a series of Propositions which the object I had in view obliged me to introduce. That object was to present the subject as a continuous chain of argument, separated from all accessories of explanation or illustration, a form which I venture to think better suited for a treatise on exact science than the semi-colloquial semi-logical form often adopted by Mathematical writers. I say 'semi-logical' advisedly, for nothing is more easy than to forget, in an argument thus interwoven with illustrative matter, what has, and what has not, been proved.

With this object in view I have introduced all such explanation and illustration as seemed necessary for a beginner, either in the form of footnotes, or, where that would have occupied too much room, of Appendices.

New words and symbols are always a most unwelcome addition to a Science, especially to one already burdened with an enormous vocabulary, yet I think the Definitions given of them will be found to justify their introduction, as the only way of avoiding tedious periphrasis. The symbols employed to represent the single elements of a Determinant, $(1\backslash 2, 1\backslash 3$, &c.$)$

require perhaps a word of apology, and it may be well to enumerate those already in use, and to point out what seem to be their chief defects.

We may commence with $\begin{Bmatrix} a_1, b_1, \ldots\ldots \\ a_2, b_2, \ldots\ldots \end{Bmatrix}$, where the change of *letter* indicates a change of *column*, and the change of *subscript* a change of *row*. Now the properties of Determinants, relating to columns, being always convertible into properties relating to rows, and vice versâ, it was a sufficient objection to this system of notation, that it represented things distinctly analogous by methods so different, and it was properly superseded by the notation introduced by Leibnitz, $\begin{Bmatrix} a_{1,1}, a_{1,2}, \ldots\ldots \\ a_{2,1}, a_{2,2}, \ldots\ldots \end{Bmatrix}$, where the changes, both of column and row, are alike denoted by subscripts. But it seems a fatal objection to this system that most of the space is occupied by a number of a's, which are wholly superfluous, while the only important part of the notation is reduced to minute subscripts, alike difficult to the writer and the reader. It was almost an obvious improvement on this system to raise the subscripts into the line, and omit the a's altogether, as suggested by Baltzer, thus — $\begin{Bmatrix} (1,1), (1,2), \ldots\ldots \\ (2,1), (2,2), \ldots\ldots \end{Bmatrix}$, and this system, though tedious for writing, might serve very well, were it not for its liability to be confused with the notation, common in Plane Algebraical Geometry, by which (1,1) denotes the Point $x=1$, $y=1$. The symbol $1\backslash 1$, which I have ventured to suggest as an emendation on this last, will be found, I have great hopes, sufficiently simple, distinct, and easy to be written. I have turned the symbol towards the left, in order to avoid all chance of confusion with $\int$, the symbol for integration.

I proceed to make a few introductory remarks on the various portions of the book, taken in order.

Chap. II. Def. I. I am aware that the word 'Matrix' is already in use to express the very meaning for which I use the word 'Block'; but surely the former word means rather the mould, or form, into which algebraical quantities may be introduced, than an actual assemblage of such quantities; for instance, $\frac{(\quad)\times(\quad)}{(\quad)}$ would deserve the name, rather than $\frac{(a+b)\times(c+d)}{(e+f)}$.

Chap. II. Def. I, VIII. Those who have read the chapters on Determinants in Mr. Todhunter's 'Theory of Equations' will notice that the meanings of the words 'Element' and 'Constituent' are here transposed: as to the former, I have only returned to Baltzer's nomenclature; and the word 'Constituent' seems to me more expressive than his word 'Term'.

Chap. III. A complete analysis of a system of simultaneous Linear Equations has always appeared to me to be a desideratum in Algebra: the subject is only touched on in Baltzer; a more complete attempt will be found in Peacock's Algebra, but I have nowhere seen anything like an exhaustive analysis. This chapter aims at furnishing this, but it has been so often altered and re-written that I put it forth at last, hoping, rather than expecting, that it will be found complete and satisfactory.

Chap. VII. This chapter will also, I hope, fulfil my aim at furnishing an *exhaustive* analysis of such properties of the Loci here considered, as can be conveniently exhibited in the form of Determinants. I had added propositions concerning the Line in Solid Geometry, but these I omit, believing that its properties are more simply investigated by other methods.

Appendix II. Section 4. This process, though extremely convenient where no ciphers, or where one or two at most, occur in the interior of a Block, nevertheless fails entirely, it must be admitted, where they occur in larger numbers: I therefore offer it merely as a fanciful addition to the processes already in use, which may in some cases lessen the labour of computation.

Appendix V. I am doubtful whether this process will ever prove of much practical use: still I think cases might arise, where in the course of a problem an algebraical function is proved to vanish, and where, by throwing it into the form of a Determinant, and so forming a set of simultaneous Equations, whose consistency depends on its vanishing, new and curious properties of the function under consideration might be evolved.

The formulæ given at the end of the book are so arranged that the student may, by covering one or more of the columns on the right hand, test for himself his knowledge of the theorems from which they are taken.

CH. CH. OXFORD,
Oct. 31, 1867.

CORRIGENDA.

P. 36. l. 10. *for*

and since, by hypothesis, $V \neq 0$, these Equations may be divided throughout by V, and written

$$x_1 = \frac{D_1}{V}, \quad -x_2 = \frac{D_2}{V};$$

read

and, dividing these Equations throughout by V,

$$x_1 = \frac{D_1}{V}, \quad -x_2 = \frac{D_2}{V};$$

and since, by hypothesis, $V \neq 0$, these values are both finite.

P. 50. l. 15. *for* $B = 0$ *read* $V = 0$.

P. 51. l. 3. *for* $\|B\| = 0$ *read* $\|V\| = 0$.

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