THE ELEMENTS OF PLANE AND SOLID GEOMETRY: WITH CHAPTERS ON MENSURATION AND MODERN GEOMETRY

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The Elements of Plane and Solid Geometry: With Chapters on Mensuration and Modern Geometry by Isaac Sharpless

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ISAAC SHARPLESS

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AND

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NOTE.

THROUGH its various stages this work has had the benefit of the suggestions and criticisms of Thomas K. Brown, Mathematical Teacher of Westtown School, whose Algebra has been prepared simultaneously with it. The authors have kept in view the object of making these books parts of a connected series of Mathematical text-books.

The following volumes are now completed:

ELEMENTARY ALGEBRA. By THOMAS K. BROWN.

ELEMENTARY PLANE GEOMETRY. By ISAAC SHARPLESS.

ELEMENTS OF PLANE AND SOLID GEOMETRY. By ISAAC SHARPLESS.

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PREFACE.

THIS little work on elementary Geometry is offered to those who desire to make themselves familiar with the general principles of the science in the limited time usually allotted to it. Beginning with simple definitions, and with postulates and axioms obvious to every one, it develops, in an unbroken series of propositions, the essential truths of Geometry. It differs alike from those treatises whose main object is to present the subject in its shortest and simplest form, and from those which are exhaustive and comprehensive in their scope.

The student does not meet at the outset, as in most modern Geometries, a collection of theorems involving constructions which he has not been taught to perform, but a simple problem; and, as he needs them, he finds other problems, so that no figure is called for which he has not the means to construct accurately and intelligently. He is impressed with the logical idea that correct conclusions can only be deduced from known premises, and he acquires practical skill in construction by applying the problems to every proposition.

While the basis of elementary Geometry is, and must ever be, contained in the works of Euclid, modern geometers, especially in France, have made important additions and corrections. Euclid's methods are sometimes cumbersome, and his omissions, especially in solid Geometry, are serious; yet years of experience attest the beneficial results of his teaching. His students think accurately and scientifically, and their training shows itself in their future work.

It has been an aim, in the preparation of this treatise, to incorporate with these advantages the improvements and additions which recent study has suggested. While thus

increasing the scope of the work, its size has been kept down by such an arrangement of the problems and theorems as secures the simplest demonstrations.

PART I. treats of Plane Geometry. The order of propositions here adopted, seems to accomplish, as fully as possible, the two ends of keeping practice always in advance of theory, and removing difficulties, as much as strict logic will permit, from the path of the beginner. These ends gained, the introduction to the science becomes interesting and suggestive. Analyses of the proof, showing at a glance the relations of the different parts of the demonstration, are given at the close of some of the propositions; thus the student has the choice of two statements, which will assist each other in giving a correct understanding of the methods. At the end of each book is a collection of exercises for original investigation and practical application. They are so explained that the average student should be able to solve them unaided—not so difficult as to cause him to give up in despair, nor so easy as to be of no interest or value. A collection of rules for the mensuration of plane surfaces, and a number of examples in them, complete Part I. This part is bound separately for the convenience of those who desire such a limited course.

PART II. contains three books, treating respectively of the geometry of planes, of solids, and of spherical geometry, with accompanying exercises. The rules for the measurement of geometrical solids are collected; a few additional rules in mensuration, not previously referred to, are proved, and numerous examples given.

PART III. contains an introduction to *Modern Geometry*, the name usually given to the discoveries in pure geometrical science, made since the advance in this direction was stayed, for a time, by the brilliant prospects opened by the Analytical Geometry of DESCARTES. For much of this part the author is indebted to the TRAITÉ DE GÉOMÉTRIE of ROUCHÉ ET DE COMBEROUSSE.

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INTRODUCTION.

EXPLANATION OF TERMS.

THE study of form is the basis of Geometry. A cubic foot of matter may be in the shape of a ball, or of a cubical block, or it may be irregular. When thus regarding only the amount of material, no attention is paid to the outline; Geometry, however, considers the outline to the exclusion of the amount of matter which it encloses.

A geometrical sphere is not a sphere of iron or wood, but a sphere of empty space. It is therefore an imaginary solid, which cannot be perceived by the senses, and for which we must use some representative, as a ball or a diagram, in order to describe it. It is a type of one class of geometrical magnitudes—**solids.**

These ideal portions of space are bounded by **surfaces** without thickness. Here, again, Geometry deals with the *form* of the surface. The surface may be flat, so that a straight-edged ruler, in whatever direction it be laid, will touch along its length, or it may be curved. It may, if flat, be limited by straight lines or curved, by lines of equal or unequal length, etc.

If, now, we suppose the edge of the ruler to be without breadth, we obtain an idea of a new kind of magnitude—a geometrical **line.** As solids are bounded by surfaces, so surfaces are bounded by lines. Lines are also imaginary, having neither breadth nor thickness. We use marks to represent