THE ELEMENTS OF ANALYTICAL MECHANICS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649144204

The elements of analytical mechanics by DeVolson Wood

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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DEVOLSON WOOD

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Trieste

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ANALYTICAL MECHANICS.

BY

DEVOLSON WOOD, A.M., C.E.,

PROFESSOR OF MATHEMATICS AND MECHANICS IN STEVENS INSTITUTE OF TECHNOLOGY; AUTHOR OF "RESISTANCE OF MATERIALS;" "ROOFS AND BRIDGES;" REVISED EDITION OF "MAHAN'S CIVIL ENGINEERING," AND "ELEMENTARY MECHANICS."

SECOND EDITION.

REVISED, CORRECTED, AND ENLARGED.

NEW YORK:
JOHN WILEY & SONS,
15 ASTOR PLACE.
1877.

Trow's
Printing and Bookbinding Co.,
Printers and Stereotypers,
205-213 *East 12th St.*,
New York.

PREFACE.

The plan of this edition is the same as the former one. It is designed especially for students who are beginning the study of Analytical Mechanics, and is preparatory to the higher works upon the same subject, and to Analytical Physics and Astronomy. The Calculus is freely used. I have sought to present the subject in such a manner as to familiarize the student with analytical processes. For this reason the solutions of problems have been treated as applications of general formulas. The solution by this method is often more lengthy than by special methods; still, it has advantages over the latter, because it establishes a uniformity in the process.

My experience has shown the importance of applying the fundamental equations to a great variety of problems. I have, therefore, in Article 24, and Chapters IV. and X., given a large number and a considerable variety of problems to be solved by the general equations under which they respectively fall.

In the revision I have been aided not only by my own experience with the use of the former edition in the class-room, but also by the friendly advice and criticism of several professors that of colleges who have used the work. The result has been several pages have been rewritten, some definitions changed, and the typographical errors corrected. Several new pages in the latter part of the work have been added. I am especially indebted to Professor E. T. Quimby, of Dartmouth College, Hanover, N. H., for his valuable suggestions and for assistance in reading the final proofs.

The *nature* of force remains as much a mystery as it was when its principles were first recognized. Of its essential nature we shall probably remain forever in ignorance. We can only deal with the *laws* of its action. These laws are determined by observing the effects produced by a force. Force is the cause of an action in the physical world. The *results* of the action may be numerous and varied. Thus, force may produce pressure, tension, cohesion, adhesion, motion, affinity, polarity, electricity, etc. Or, to speak more properly, since force may be transmuted from one state to another, we would say that the above terms are names for the different manifestations of force.

The question "what is the correct measure of force" has taken different phases at different times. During the last century it was contended by some that momentum (Mv) was the correct measure, while others contended that it should be the work which it can do in a unit of time ($\frac{1}{2}Mv^2$). But as one has happily expressed it, "theirs was only a war of words;" for the real measure of force enters only as a factor in the expressions. Thus, if F be a constant force, the value of the momentum is Ft, see page 51, and of the work Fs, see page 45. At the present day some contend that the only measure of force is the motion which it produces, or would produce, in a unit of time. This is called the ABSOLUTE MEASURE, and THE ABSOLUTE UNIT OF FORCE *is the velocity which the force produces, or would produce, in a unit of mass in a unit of time if it acted during the unit with the intensity which it had at the instant considered.* If the intensity of the force were constant, the velocity which it produced at the end of the unit of time would be the required velocity. Hence, the absolute measure of any force acting on any mass is *the product of the mass into the acceleration;* and is the second member of equation (21). This is a correct measure, and is accepted as such by all writers on mechanics.

But those who contend that this is the only measure, necessarily deny that *weight*, or more generally *pounds*, is a measure. I contend that *pounds* is *a* measure of the intensity of a force both statically and dynamically. Many authors maintain the same position. Indeed, it is probable that the position which I have taken can be *deduced* from any standard work on mechanics; but in some it is left to inference. Thus, in Smith's *Mechanics*, page 1, we find this terse and correct definition, "The *intensity* of a force may be measured, statically, by the pressure it will produce; dynamically, by the quantity of motion it will produce." I say this is correct, but I will add that the intensity of a force which produces a given motion is also measured by a pressure, or by something equivalent to a pressure, or to a pull. To those who will look at it analytically, it is only necessary to say that the first member of equation (21) is measured in *pounds*. If we know the absolute measure, we may easily find its value in *pounds*.

The *pound* here referred to is the result of the action of gravity upon a certain quantity of matter. The amount of matter having been fixed, either by a legal enactment or by common consent, and declared to be one pound at a certain place, its weight, as determined by a standard spring-balance at any other place, becomes a measure of the force of gravity as compared with the fixed place. This standard spring-balance may measure the intensity in pounds of any other force, whether the body upon which the force acts be at rest or in motion. If a perfectly free body were placed in a hollow space at the centre of the earth, at which place it would be devoid of weight, and pulled or pushed by a constant force, whose intensity, measured by a standard spring-balance, equaled the weight of an equal body on the surface of the earth, then would its motion be the same as that of a falling body. See page 24, Problem 7. In the forces of nature producing motion, there being no visible connection between the point of action of the force and the body upon which it acts, we are unable to *weigh* their intensity except by calculation. If the absolute measure is known, the *pounds* of intensity may be computed. The absolute measure of the force of gravity on a mass m is mg, and the weight of the body being W, we have $W = mg$. The sun acts upon the earth with a force which may be expressed by the absolute measure, and also by a certain number of pounds of force. More than half of the examples in Article 24 involve an equality between *pounds of intensity* and the absolute measure of the force. The fact is, that, in case of motion, these quantities are co-relative. Since, then, it is correct to use the term *pound* as the measure of the intensity of a force whether the body be at rest or in motion, and since it is in common use, and the student is familiar with it, I prefer to consider a force as measured by a certain number of pounds. See Article 9. It is more simple, containing as it does only one element, than the absolute measure, which contains three elements—mass, velocity, and time.

There is another advantage in thus measuring force. Students frequently, and in some cases writers, use the expressions, "quantity of force," "amount of force," "force of a blow," etc., when they mean (or should mean) momentum, or work, or vis viva. In such cases an attempt to answer the question "how many pounds of force" would show at once that the quantity referred to was not *force*.

So much ambiguity, or at least indefiniteness, has arisen in regard to the term force, that I have rejected the terms "Impulsive Force" and "Instantaneous Force," and used the term "Impulse" instead of them. We know nothing of an instantaneous force, that is, one which requires no time for its action. I also reject the expression *force of inertia*. I do not believe that *inertia* is a *force*. To the question "The inertia of a body is how many pounds of force" there is no answer.

The term *moment of inertia* has no physical representation. The nearest approach to it is in the expression for the vis viva of a rotating body. In such problems the moment of inertia forms an important factor. The energy of a rotating body having a constant angular velocity is directly proportional to its moment of inertia in reference to its axis of rotation. See page 202. But *motion* is not necessary for its existance. See page 165. The expression appears in the discussion of numerous statical problems, such as the flexure of a beam, the centre of pressure of a fluid, the centre of gravity of certain solids, etc. It is not the moment of a moment, although it may be so construed as to appear to be of that *form*. Some other term might be more appropriate. Even the expression *moment of the mass* would be less objectionable.

The subjects of *Centrifugal Force* and *Unbalanced Force* have been discussed of late in *Engineering*. Some assert that there is no such thing as a centrifugal force. Much unprofitable discussion may be avoided by strictly defining the terms used. If it is defined to be a force equal and opposite to the deflecting force, it will, at least, have an ideal existence, just as the resultant in statical problems has an ideal existence. But the vital question is, is the centrifugal force active when the deflecting force acts? Or, in other words, do both act upon the body at the same time? It seems, however, quite evident that if both acted upon the body at the same time they would neutralize each other, and the body would move in a straight line. Hence, in the movement of the planets, or of any free rotating body, there is no centrifugal force. But in the case of a locomotive running around a curve there may be both centripetal and centrifugal forces; the former acting against the locomotive to force it away from a tangent to the track; the latter, against the track, tending to force it outward. Wherever the force is conceived to act, whether just between the rail and wheel or at some other point, it is evident that both do not act upon the same body.

Similarly in regard to the *unbalanced force*. It is a convenient term to use, but, in a strict sense, an unbalanced force does not exist; for action and reaction are equal and opposite. But in reference to a particular body, all other conditions being ignored, the force may be unbalanced. Thus, when a ball is fired from a cannon, the force of the powder, considered in the direction of the motion of the ball only, is unbalanced; but the powder exerts an equal force in the opposite direction, and in that sense also is unbalanced. But when the entire effect of the