

**ELEMENTS OF THE
PRECISION OF
MEASUREMENTS AND
GRAPHICAL METHODS**

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Elements of the Precision of Measurements and Graphical Methods by H. M. Goodwin

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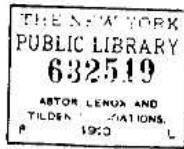
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PREFACE.

In its present form the "Elements of the Precision of Measurements and Graphical Methods" represents the ground covered in a brief course which has been given for a number of years at the Massachusetts Institute of Technology to all students in connection with their work in the Physical Laboratory. The author has been induced to amplify the printed "Notes" on this subject and give them a wider circulation in response to repeated requests to use them elsewhere. Although prepared primarily to meet the needs of his own classes, it is hoped, in the present form, they may prove useful in other technical schools and colleges where quantitative work forms a part of the curriculum, and also to engineers whose work involves experimental testing. In many laboratories far too little weight is attached to the discussion of the magnitude and effect of sources of error on a result. This has been forced upon the writer's attention as the result of personal interviews with hundreds of graduate students entering the Institute, who apply for excuse from laboratory work. It is the exceptional student who has any conception how to figure out the precision of a final computed result from the precision of his individual measurements, and this is true even though his laboratory note-book shows his work to have been carefully and creditably performed. It is the author's firm conviction that one of the most valuable and enduring benefits of physical laboratory training to a student of Science or Engineering is the acquisition of the proper view-point with which to approach an investigation, be it either purely scientific or technical; that is, the ability to recognize the essentials of a problem at the outset, so as to economize both time and labor in its solution. Although the exercise of judgment, based upon the personal experience of the investigator, is essential to the "best solution" of any experimental problem, still it is desirable to direct the student's attention to precision methods at an early stage of his laboratory work. Experiments have shown that this may be satisfactorily done as soon as he has had a little practice in exact measurements and can handle the elements of Differential Calculus. At the Institute the course is given at the middle of the sophomore year, after the student has performed some six or eight experiments on fundamental measurements in Mechanics. Continued application of the principles is then made in subsequent laboratory work throughout the junior and senior years, and a precision discussion is regarded as

an important feature of the final thesis. It has been the writer's experience that students have little trouble in understanding the general principles involved, but meet with considerable difficulty in applying these principles to concrete problems. For this reason the subject is most satisfactorily taught to small sections by recitations based on the text and the solution of numerous problems selected from the book and from the current laboratory work. A close correlation of class-room and laboratory work is indeed highly desirable, and in the Rogers Laboratory of Physics it is the practice to require with each laboratory report a precision discussion of the data or a solution of some precision problem related to the experiment. The laboratory manuals have been written with this in view.

The method of treatment has been kept as brief as possible. A full discussion of the subject, with proofs based on the Theory of Probability and the Method of Least Squares, would so enlarge the work as to defeat its end. Proofs of the few theorems and formulæ which the student is asked to assume may be found in any good treatise on Least Squares. An excellent treatment is that given in Bartlett's "Method of Least Squares." A more exhaustive treatment of Precision Methods may be found in Holman's "Precision of Measurements."

A chapter on the solution of illustrative problems has been added to assist students who find it necessary to work up the subject by themselves. The collection of problems has been compiled from recent examination papers. The chapter on Graphical Methods contains specific directions for constructing graphs, and general directions for obtaining therefrom the functional relationship between two variables. For engineering students, as well as physicists, the method of logarithmic plotting will be found of wide application. In the Appendix several tables, of assistance in precision computations, have been added.

In conclusion the author desires to express his indebtedness for many suggestions to his colleagues who have so ably assisted him in the instruction of this subject in recent years, and in particular to Professor William J. Disko, whose experience in teaching this and related subjects has been most helpful.

H. M. GOODWIN.

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PRECISION OF MEASUREMENTS.

Classification of Physical Measurements.—All physical measurements may be classed as direct or indirect according as the measurement gives the desired result directly, or as the result is obtained by combining the results of several measurements by means of some formula. Examples of the first class are the measurement of a length by means of a scale, the mass of a body by means of an equal arm balance, and the electrical resistance of a wire by the direct method of substitution. Examples of indirect measurements are the determination of g , the acceleration due to gravity, by means of a pendulum, involving the measurement of the length and time of vibration of the pendulum, the determination of the index of refraction of a substance from measurements of the angle and the minimum deviation of a prism by means of a spectrometer, and the determination of the specific heat of a substance by the method of mixtures in which the results of the measurement of a number of temperatures and weights are combined. The great majority of problems arising in practice come under the second class.

Reliability of a Result.—In order that the result of any measurement, whether direct or indirect, may be of any scientific or technical value, it is necessary to have some numerical estimate or measure of its reliability. The importance of such a measure cannot be overestimated. The result of a test, of a study of an instrument or method, or of the determination of a constant, may be rendered almost worthless, unless the investigator is able to state the degree of reliance which can be placed upon it. This phase of an investigation should be kept constantly in mind in all laboratory work. The student's ability to intelligently dis-